Forecasting the Equity Risk Premium: Predictability versus Profitability

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Motivation

- Forecasting next month's ERP, using a univariate linear regression framework
- Long list of macroeconomic predictors:
  T-bill rate (Fama & Schwert, JFE-1977), dividend yield (Fama & French, JFE-1988), D/P (Campbell & Shiller, RFS-1988), term spread (Keim & Stambaugh, JFE-1986), volatility (French, Schwert & Stambaugh, JFE-1987), B/M (Kothari & Shanken, JFE-1997), inflation (Fama & Schwert, JFE-1977), default spread (Fama, JFE-1986), corporate issuing activity (Baker & Wurgler, JF-2000), consumption-surplus ratio (Campbell & Cochrane, JPE-1999), consumption-wealth ratio (Lettau & Ludvigson, JF-2001)
- Broadly speaking, these have "good" in-sample forecasting power.
- Out-of-sample, they struggle to beat historical average (Goyal and Welch, RFS-2008)
- Significant implications for asset allocation decisions

Motivation

- Following Goyal and Welch (RFS-2008), two streams of literature have emerged:
  - Devise/identify "better" forecasting variables:

    output gap (Cooper & Priestley, RFS-2009), VRP (Bollerslev et al. RFS-2009), 52-week high vs. historical high (Li & Yu, JFE-2012), aggregate ICC (Li, Ng & Swaminathan, JFE-2013), technical indicators (Neely, Rapach, Tu & Zhou MS-2014), economic growth (Mller & Rangvid, JFE-2015), investor sentiment (Huang, Jiang, Tu & Zhou, RFS-2015), mean-reversion (Huang, Jiang, Tu & Zhou, WP-2015), short interest (Rapach, Ringgenberg & Zhou, JFE-2015)
  - Refine the forecasting model - OUR FOCUS!
- Campbell & Thompson (RFS-2008) and Pettenuzzo, Timmermann & Valkanov (JFE-2014) introduce economic constraints to generate non-negative ERP forecasts
- Argument: positive ERP forecast justifies equity investing.
- Research questions:
  - How do constrained models perform in months/periods of negative ERP realisations?
  - How do constrained models perform in periods of high volatility?
  - What are the asset allocation implications from using constrained ERP models?
  - What are the implications of real-life investment constraints?

Main Findings - Contributions

- Out-of-sample forecasting performance: the constrained models ...
  - ...outperform OLS across the entire sample period (1947-2013)
  - ...outperform OLS during periods of positive ERP realisations
  - ...outperform OLS during expansionary periods
  - ...underperform OLS during periods of negative ERP realisations
  - ...underperform OLS during recessionary periods
  - ...underperform OLS during periods of high volatility

- Asset allocation implications:
  - Constrained models generate CER gains and losses against OLS as above.
  - Any economic benefits of the constrained models diminish for conservative investors.
  - Short-sale constraints or even a minimum required investment is equivalent to a constrained model.
  - No-leverage constraints become more often binding for constrained models and therefore reduce any economic benefit during up markets.

- Our findings have important implications for designing new ERP forecasting models.

Out-of-sample ERP Forecasting Models

- Traditional linear regression forecasting model:

$$OLS: \quad \hat{r}^{OLS}_{t+1|t} = \hat{\alpha} + \hat{\beta} \cdot x_t$$

where $\hat{\alpha}$ and $\hat{\beta}$ are estimated using information up to time t:

$$r_{\tau+1} = \alpha + \beta \cdot x_\tau + \epsilon_{\tau+1}, \quad \tau = 1, \cdots, t-1$$

- Campbell & Thompson (RFS-2008, CT) truncation model:

$$CT: \quad \hat{r}^{CT}_{t+1|t} = max\left(\hat{\alpha} + \hat{\beta} \cdot x_t, 0\right)$$

- Pettenuzzo, Timmermann & Valkanov (JFE-2014, PTV) Bayesian model:

$$PTV: \quad \hat{r}^{PTV}_{t+1|t} = \bar{\alpha} + \bar{\beta} \cdot x_t$$

where $\bar{\alpha}$ and $\bar{\beta}$ are the averages of all pairs of values (α, β) that belong to a set $\mathcal{A}_t$:

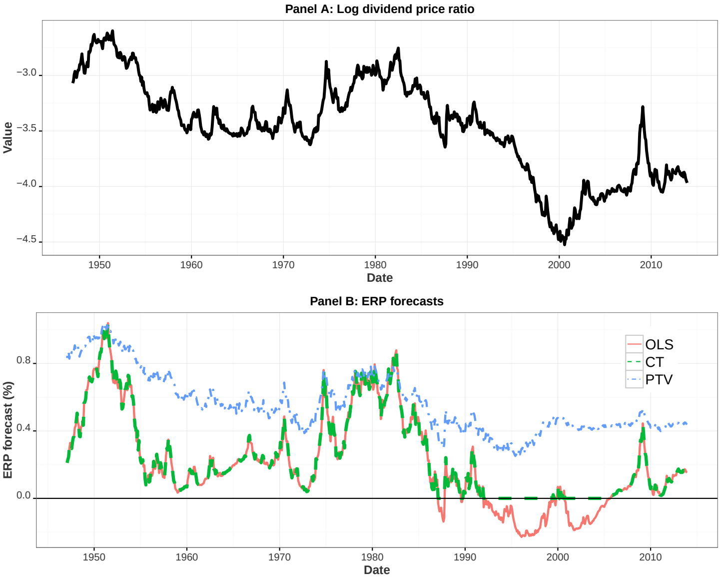
$$\mathcal{A}_t = \{(\alpha, \beta) : \alpha + \beta \cdot x_\tau \geqslant 0, \quad \forall \tau = 1, \cdots, t\}$$

Data

- Equity Risk Premium (ERP):
  - S&P500 monthly excess total (i.e. including dividends) logarithmic return
  - Source: CRSP
- Forecasting variables:
  - Log dividend-price ratio
  - Source: webpage of Amit Goyal
- Sample period:
  - Full sample: January 1927 - December 2013
  - Training period: 20 years
  - Out-of-sample: January 1947 - December 2013 (804 months)

Sample: Jan. 1927 - Dec. 2013	Mean	St.Dev	Skewness	Kurtosis	AC(1)
Annualised Log Excess Returns in %	5.90	19.15	-0.42	10.58	0.09
Dividend Price Ratio (d/p)	-3.35	0.46	-0.32	2.84	0.99

January 1927 – December 2013 - Out-of-sample ERP forecasts (initial training period: 20 years)



Evaluating OOS forecasting accuracy

Estimate the Difference in the Cumulative Sum of Squared Errors (DCSSE) between a benchmark model and the model of interest (Goyal & Welch, MS-2003):

- Typical benchmark - Historical Average (HA):

$$DCSSE_{HA} = \sum_{\tau=t_{OOS}}^{T} \left(r_\tau - \hat{r}^{HA}_{\tau|\tau-1}\right)^2 - \sum_{\tau=t_{OOS}}^{T} \left(r_\tau - \hat{r}^{Model}_{\tau|\tau-1}\right)^2$$

  where $Model = \{OLS, CT, PTV\}$; $\tau = t_{OOS}$: first OOS forecast; T: end of sample.

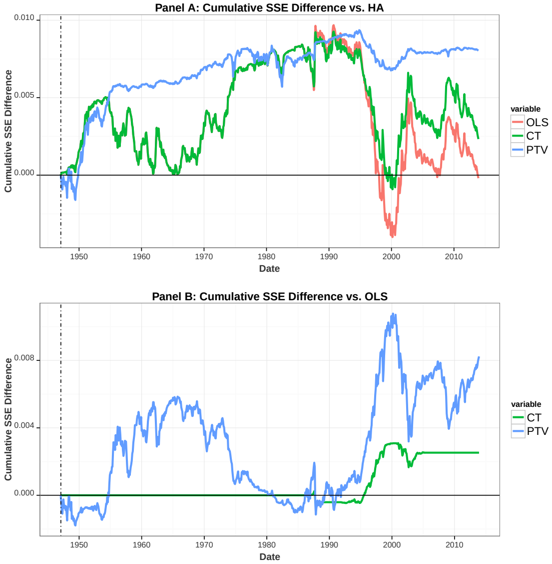
  - Upward movement: model is more accurate than HA
  - Downward movement: model is less accurate than HA
  - Flat: the model and the HA are equally good (or bad)

- Our focus is against OLS:

$$DCSSE_{OLS} = \sum_{\tau=t_{OOS}}^{T} \left(r_\tau - \hat{r}^{OLS}_{\tau|\tau-1}\right)^2 - \sum_{\tau=t_{OOS}}^{T} \left(r_\tau - \hat{r}^{Model}_{\tau|\tau-1}\right)^2$$

  where $Model = \{CT, PTV\}$.

Forecasting accuracy against the historical average & OLS



Forecasting accuracy against OLS

- Proportional decrease in MSFE between the model of interest and the benchmark (OLS):

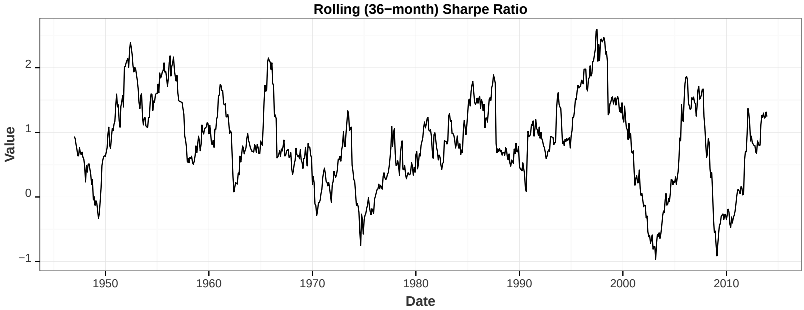
$$R^2_{OOS,OLS} = 1 - \frac{MSFE_{Model}}{MSFE_{OLS}} = 1 - \frac{\sum_{\tau=t_{OOS}}^{T}\left(r_\tau - \hat{r}^{Model}_{\tau|\tau-1}\right)^2}{\sum_{\tau=t_{OOS}}^{T}\left(r_\tau - \hat{r}^{OLS}_{\tau|\tau-1}\right)^2}$$

where $Model = \{CT, PTV\}$.

OOS Period: Jan. 1947 - Dec. 2013		d/p
	# months	804
RMSFE x 100	OLS	4.227
	CT	4.223
	PTV	4.215
$R^2_{OOS,OLS}$	CT	0.18%**
	PTV	0.57%***
% of months with smaller error vs. OLS	CT	13.20%
	PTV	55.54%

- The constrained models generally outperform the unconstrained OLS model.

36-month rolling Sharpe ratio of S&P500



- The constrained models outperform the OLS model during strong market rallies
- However, they underperform during volatile periods of negative ERP realisations
- We focus on regime dependent analysis:
  - Return dependent analysis (up versus down)
  - Volatility dependent analysis (high versus low volatility)

Up versus Down markets

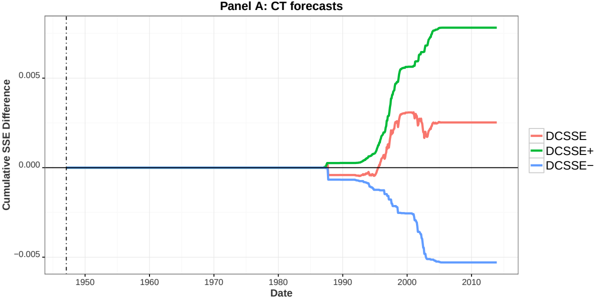
Split $DCSSE_{OLS}$ calculation separately for months with positive and negative EPR realisations:

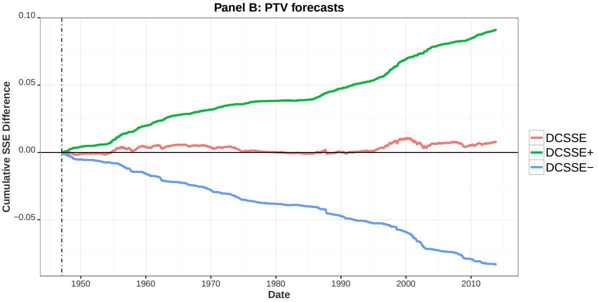
$$DCSSE^{+}_{OLS} = \sum_{\tau=t_{OOS}}^{T} \left(r_\tau - \hat{r}^{OLS}_{\tau|\tau-1}\right)^2 \cdot \mathcal{I}_{r_\tau \geqslant 0} - \sum_{\tau=t_{OOS}}^{T} \left(r_\tau - \hat{r}^{Model}_{\tau|\tau-1}\right)^2 \cdot \mathcal{I}_{r_\tau \geqslant 0}$$

$$DCSSE^{-}_{OLS} = \sum_{\tau=t_{OOS}}^{T} \left(r_\tau - \hat{r}^{OLS}_{\tau|\tau-1}\right)^2 \cdot \mathcal{I}_{r_\tau < 0} - \sum_{\tau=t_{OOS}}^{T} \left(r_\tau - \hat{r}^{Model}_{\tau|\tau-1}\right)^2 \cdot \mathcal{I}_{r_\tau < 0}$$

where $I_{r_\tau \geqslant 0}$ and $I_{r_\tau < 0}$ denote indicator functions for positive and negative ERP realisations.

Up versus Down markets: using D/P ratio





Up versus Down markets: the statistics

		All	Down	Up
	# months	804	328	476
RMSFE x 100	OLS	4.227	4.746	3.828
	CT	4.223	4.763	3.806
	PTV	4.215	5.005	3.568
$R^2_{OOS,OLS}$	CT	0.18%**	-0.72%	1.12%***
	PTV	0.57%***	-11.25%	13.12%***
% of months with smaller error vs. OLS	CT	13.20%	0.30%	22.11%
	PTV	55.54%	1.52%	92.84%

Up versus Down markets: further statistics

		NL	NS	PS	PL
	# months	164	164	238	238
ERP realisations	min	-25.0%	-2.4%	0.0%	2.9%
	max	-2.4%	0.0%	2.9%	15.0%
RMSFE x 100	OLS	6.512	1.625	1.476	5.213
	CT	6.533	1.638	1.451	5.188
	PTV	6.813	1.921	1.187	4.909
$R^2_{OOS,OLS}$	CT	-0.66%	-1.64%	3.37%***	0.94%***
	PTV	-9.47%	-39.71%	35.33%***	11.33%***
% of months with smaller error vs. OLS	CT	0.00%	0.61%	23.95%	20.25%
	PTV	2.44%	0.61%	88.24%	97.47%

Recessions versus Expansions

- Split the OOS period in recessionary and expansionary months based on NBER:

- Split the OOS period in months of low volatility, medium volatility and high volatility:

- Based on S&P500 monthly realised volatility (sum of squared daily returns)
- Low-medium breakpoint: 9.71%
- Medium-high breakpoint: 13.92%

		Business Cycles		Volatility Regimes		
		Rec	Exp	Low	Med	High
	# months	122	682	268	268	268
RMSFE x 100	OLS	5.774	3.885	2.848	3.609	5.696
	CT	5.777	3.879	2.838	3.602	5.697
	PTV	5.855	3.848	2.702	3.587	5.753
$R^2_{OOS,OLS}$	CT	-0.13%	0.30%***	0.70%***	0.39%***	-0.04%
	PTV	-2.82%	1.92%***	9.98%***	1.23%**	-2.04%
% of months with smaller error vs. OLS	CT	2.46%	15.12%	11.19%	13.11%	15.30%
	PTV	43.44%	57.71%	66.04%	54.68%	45.90%

Asset Allocation Implications

A risk-averse investor allocates between the equity market and a risk-free asset:

- Mean-variance preferences suggest an equity allocation of:

$$w_t = \frac{1}{\gamma} \cdot \frac{\hat{r}_{t+1|t}}{\hat{\sigma}^2_{t+1|t}}$$

  - γ: relative risk aversion
  - $\hat{r}_{t+1|t}$: OOS ERP forecast based on a forecasting model
  - $\hat{\sigma}^2_{t+1|t}$: ERP forecast variance; assume a rolling five-year historical estimate

- Typically the equity weighted is constrained (the very big majority of papers...):

$$w_t = \max\left\{\min\left[\frac{1}{\gamma} \cdot \frac{\hat{r}_{t+1|t}}{\hat{\sigma}^2_{t+1|t}}, w_{max}\right], w_{min}\right\}$$

- Baseline scenario: $\gamma = 3$, $w_{min} = -0.5$ and $w_{max} = 1$ (i.e. no leverage)

Asset Allocation Implications

- Portfolio return:

$$r_{p,t+1} = w_t \cdot r_{t+1} + r_{f,t+1}$$

  - r_{t+1}: next month's equity excess return
  - $r_{f,t+1}$: risk-free rate prevailing at the end of month t

- Economic benefit of ERP forecasting models is measured by CER:

$$CER = \hat{\mu}_P - \frac{\gamma}{2} \cdot \hat{\sigma}_P^2$$

  - $\hat{\mu}_P$: average annual portfolio return over the OOS period
  - $\hat{\sigma}_P^2$: annual portfolio variance over the OOS period

- CER can be interpreted as the annual fee that the investor would be willing to pay to have access to the respective ERP forecast and therefore to exploit it.

- Relative economic benefit of constrained models vs. OLS:

$$CER\ gain\ vs.\ OLS = CER_{Model} - CER_{OLS}$$

where $Model = \{CT, PTV\}$.

CER Gains

Baseline parameter values: $\gamma = 3$, $w_{min} = -0.5$, $w_{max} = 1$			
Forecast Model	CER gain	Volatility	Sharpe ratio
Panel A: Overall			
CT	0.52%	8.82%	0.79
PTV	1.80%	12.86%	0.74
Panel B: Down markets			
CT	-1.66%	7.38%	-1.84
PTV	-19.02%	9.56%	-3.18
Panel C: Up markets			
CT	2.16%	7.28%	2.91
PTV	18.02%	7.82%	4.76
Panel D: Recessions			
CT	-0.33%	15.76%	0.06
PTV	-6.70%	18.88%	-0.20
Panel E: Expansions			
CT	0.67%	6.86%	1.17
PTV	3.34%	11.34%	1.05

Sensitivity Analysis

Maximum and minimum weights:

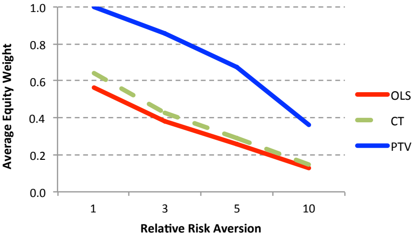
- Campbell & Thompson (RFS-2008), Goyal & Welch (RFS-2008), Rapach, Strauss & Zhou (RFS-2010), Neely, Rapach, Tu & Zhou (MS-2014), Huang, Jiang, Tu, & Zhou (RFS-2015): $w_{min} = 0$ and $w_{max} = 1.5$
- Huang, Jiang, Tu & Zhou (WP-2015): $w_{min} = -0.5$ and $w_{max} = 1$
- Rapach, Ringgenberg & Zhou (JFE-2015): $w_{min} = -0.5$ and $w_{max} = 1.5$
- Ferreira & Santa-Clara (JFE-2011), Pettenuzzo et al. (JFE-2014): no restrictions

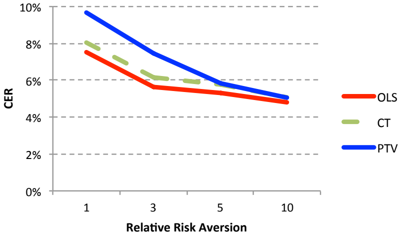
The typical level of relative risk aversion that is used in these studies is $\gamma = 3$, except:

- Ferreira & Santa-Clara (JFE-2011) use $\gamma = 2$
- Pettenuzzo, Timmermann & Valkanov (JFE-2014) present results for $\gamma = 2, 5, 10$
- Huang, Jiang, Tu & Zhou (RFS-2015) present results for $\gamma = 1, 3, 5$

The effect of risk aversion

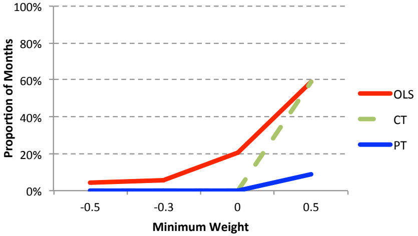
- Assume $w_{min} = -0.5$ and $w_{max} = 1$
- Average equity weight & CER for different levels of γ (1, 3, 5, 10):





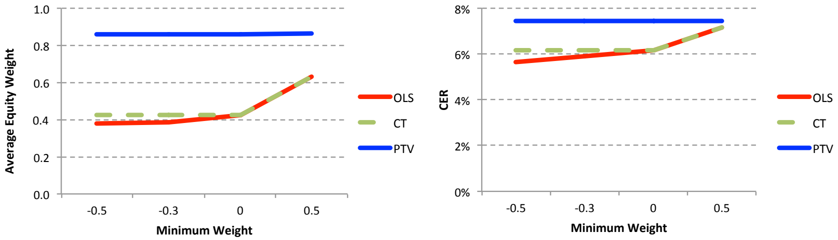
The effect of minimum weight and short selling

- Assume $\gamma = 3$ and $w_{max} = 1$.
- Different levels of the w_{min}:
  - -0.5 (allow short selling of up to 50% of the total wealth)
  - -0.3 (allow short selling of up to 30% of the total wealth)
  - 0 (allow no short selling)
  - 0.5 (require minimum equity investment of at least 50% of the total wealth)
- Minimum weight constraint becoming binding:



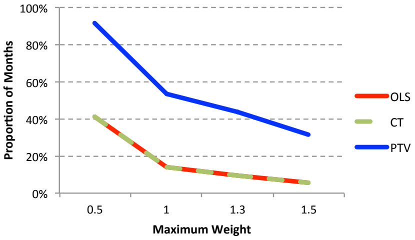
The effect of minimum weight and short selling

- Average equity weight & CER for different levels of w_{min}:



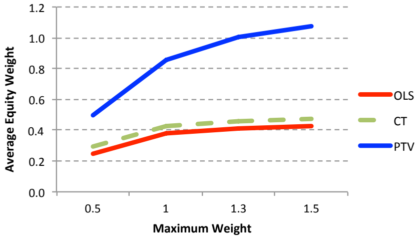
The effect of maximum weight and leverage

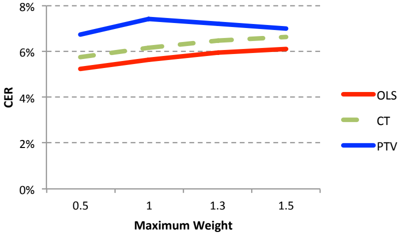
- Assume $\gamma = 3$ and $w_{min} = -0.5$.
- Different levels of the w_{max}:
  - 0.5 (constrain the portfolio against an all-equity possibility)
  - 1 (allow no leverage)
  - 1.3 (allow leverage of 30% of the total wealth)
  - 1.5 (allow leverage of 50% of the total wealth)
- Maximum weight constraint becoming binding:



The effect of maximum weight and leverage

- Average equity weight & CER for different levels of w_{max}:





Conclusions and Future Research

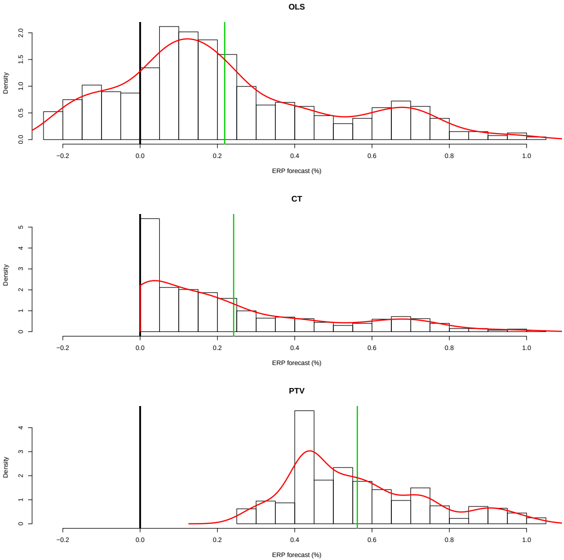
To summarise:

- Our findings expose an important pitfall of constrained ERP forecasting models
- Constrained ERP forecasting models generally outperform
- However, they struggle when it matters the most...
- ... negative ERP realisations, recessions, high-volatility periods
- Important asset allocation implications for actual dynamic portfolio decisions
- Conservative and short-sale / leverage constrained investors cannot benefit generally from constrained models

Future research:

- New ERP models should incorporate regime dependence
- The first attempt towards this: Huang, Jiang, Tu, & Zhou (WP-2015)
- Suggestion: dynamic combination of unconstrained and constrained models
- New forecasting variables: e.g. option implied metrics

Appendix


OLS
Density
0.0
0.5
1.0
1.5
2.0
−0.2
0.0
0.2
0.4
0.6
0.8
1.0
ERP forecast (%)
CT
Density
0
1
2
3
4
5
−0.2
0.0
0.2
0.4
0.6
0.8
1.0
ERP forecast (%)
PTV
Density
0
1
2
3
4
−0.2
0.0
0.2
0.4
0.6
0.8
1.0
ERP forecast (%)