Puzzles in Index Option Returns

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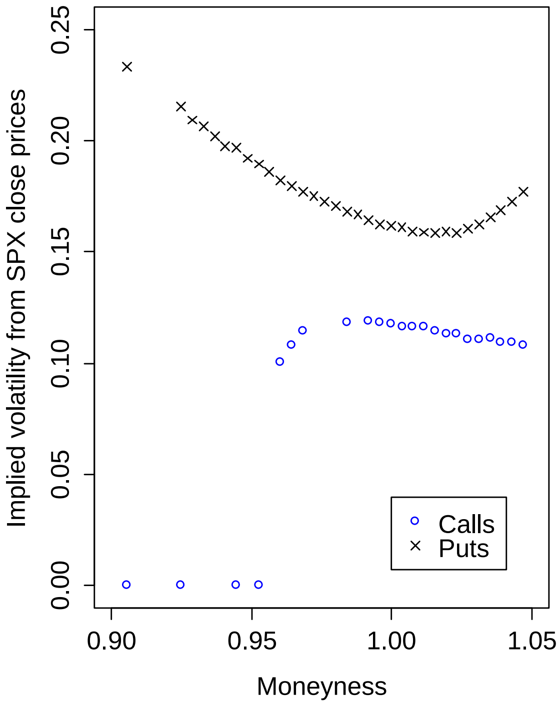
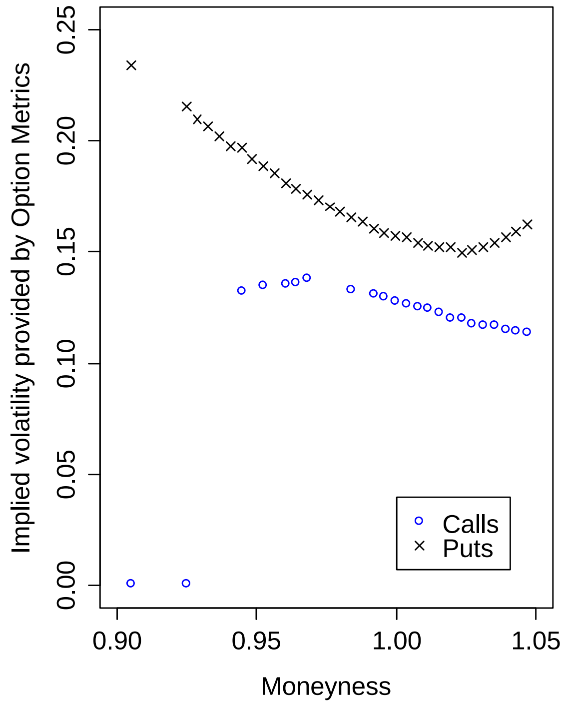
Index option returns: still puzzling

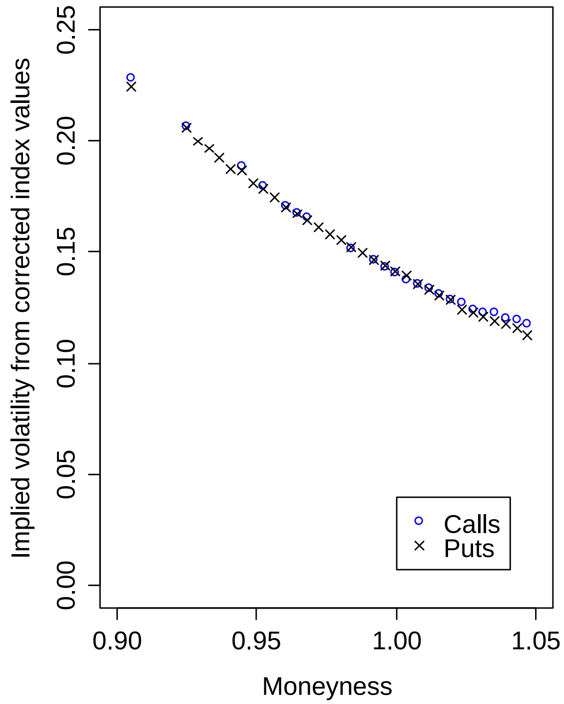
1. Are options mispriced with respect to theoretical pricing bounds?
2. Why are options so expensive?
3. Is the smile too steep?
4. Why is the pricing kernel implied in option prices typically hump-shaped?
5. Why is the risk-neutral distribution much more skewed than the objective distribution?

Matching index level and option price

S&P 500 options on May 17, 2006







Index level: 1270.32

Option Metrics

Index level: 1264.10

Price bounds without transaction costs

$$\bar{c}(S_t,t) = \frac{\mathrm{E}[(S_T-K)^+ \mid S_t]}{R_S^{T-t}}$$

$$\underline{c}(S_t,t) = \frac{S_t}{(1+d)^{T-t}} - \frac{K}{R^{T-t}} + \left[\frac{\mathrm{E}[(K-S_T)^+ \mid S_t]}{R_S^{T-t}} \right]$$

$$\bar{p}(S_t,t) = \bar{c}(S_t,t) + \frac{K}{R^{T-t}} - \frac{S_t}{(1+d)^{T-t}}$$

$$\underline{p}(S_t,t) = \underline{c}(S_t,t) + \frac{K}{R^{T-t}} - \frac{S_t}{(1+d)^{T-t}}$$

$$\bar{c}(S_t,t) - \underline{c}(S_t,t)$$

$$= \bar{p}(S_t,t) - \underline{p}(S_t,t) = \frac{K}{R^{T-t}} - \frac{K}{R_S^{T-t}}$$

Derivation of an upper call bound

- Initial position: 1 share of stock with price S_0
- Restriction: final wealth non-decreasing in the final stock price S
- Assumption: risk-averse investor (concave utility function)
- Consider the zero-net-cost position
  - Short position in 1 call with strike K
  - Long position in C/S_0 shares of the stock
- Payoff at the terminal date:

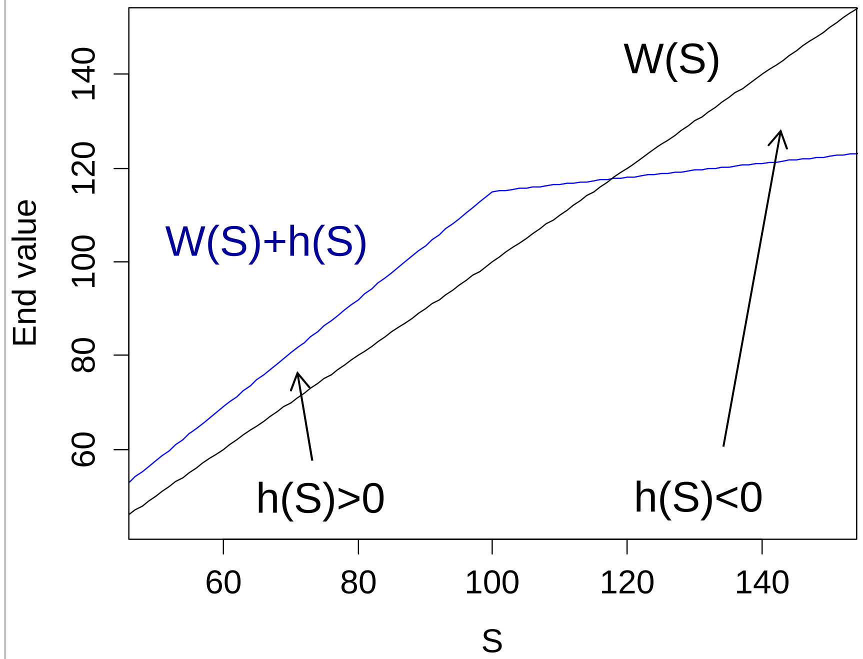
$$h(S) = CS/S_0 - [S-K]^+$$

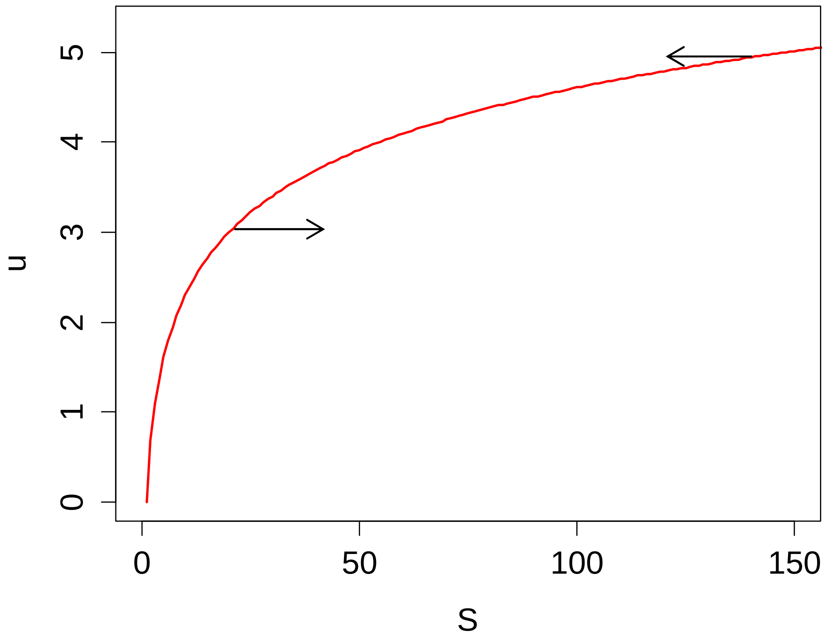
$$\begin{aligned} W(S) + h(S) &= S + CS/S_0 - [S-K]^+ \\ &= \begin{cases} CS/S_0 + K & \text{for} \quad S > K \\ S(1 + C/S_0) & \text{for} \quad S \le K \end{cases} \end{aligned}$$

Illustration

$$E[h(S)] > 0 \Rightarrow E[u(W(S)+h(S))] - E[u(W(S))] > 0$$

$$E[h(S)] \leq 0 \Leftrightarrow C \leq \frac{S_0}{E[S]} E[[S-K]^+]$$





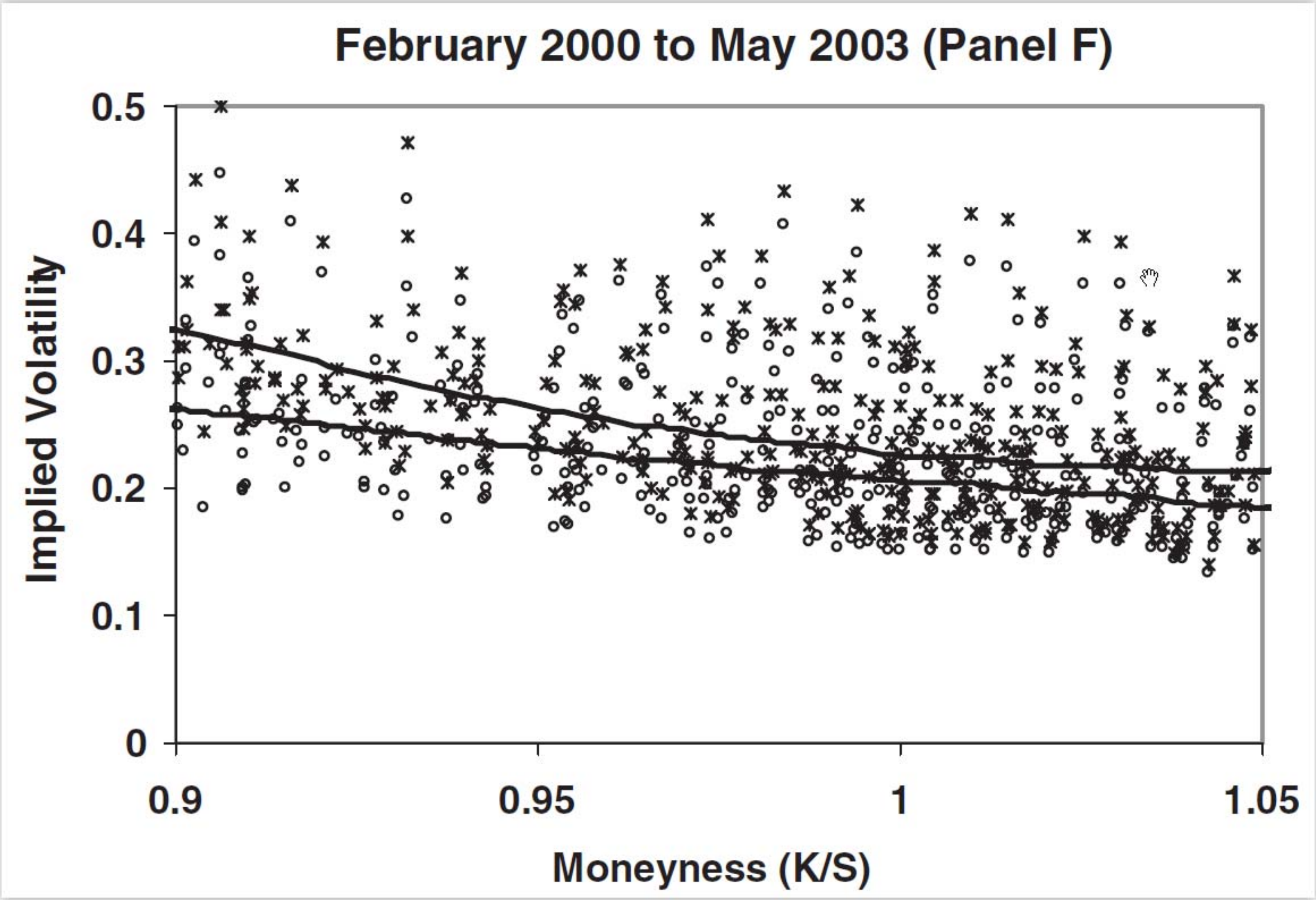
Constantinides et al. (2009, 2011)

Widespread violations of stochastic dominance by 1-month S&P 500 index call options over 1986–2006 imply that a trader can improve expected utility by engaging in a zero-net-cost trade net of transaction costs and bid-ask spread.

Most of the violations are violations of the upper bound. The decrease in violations over the 1988–1995 postcrash period (panels B–D) is followed by a substantial increase in violations over 1997–2003 (panels E and F). This is a novel finding and casts doubts on the hypothesis that the options market is becoming more rational over time, particularly after the crash.

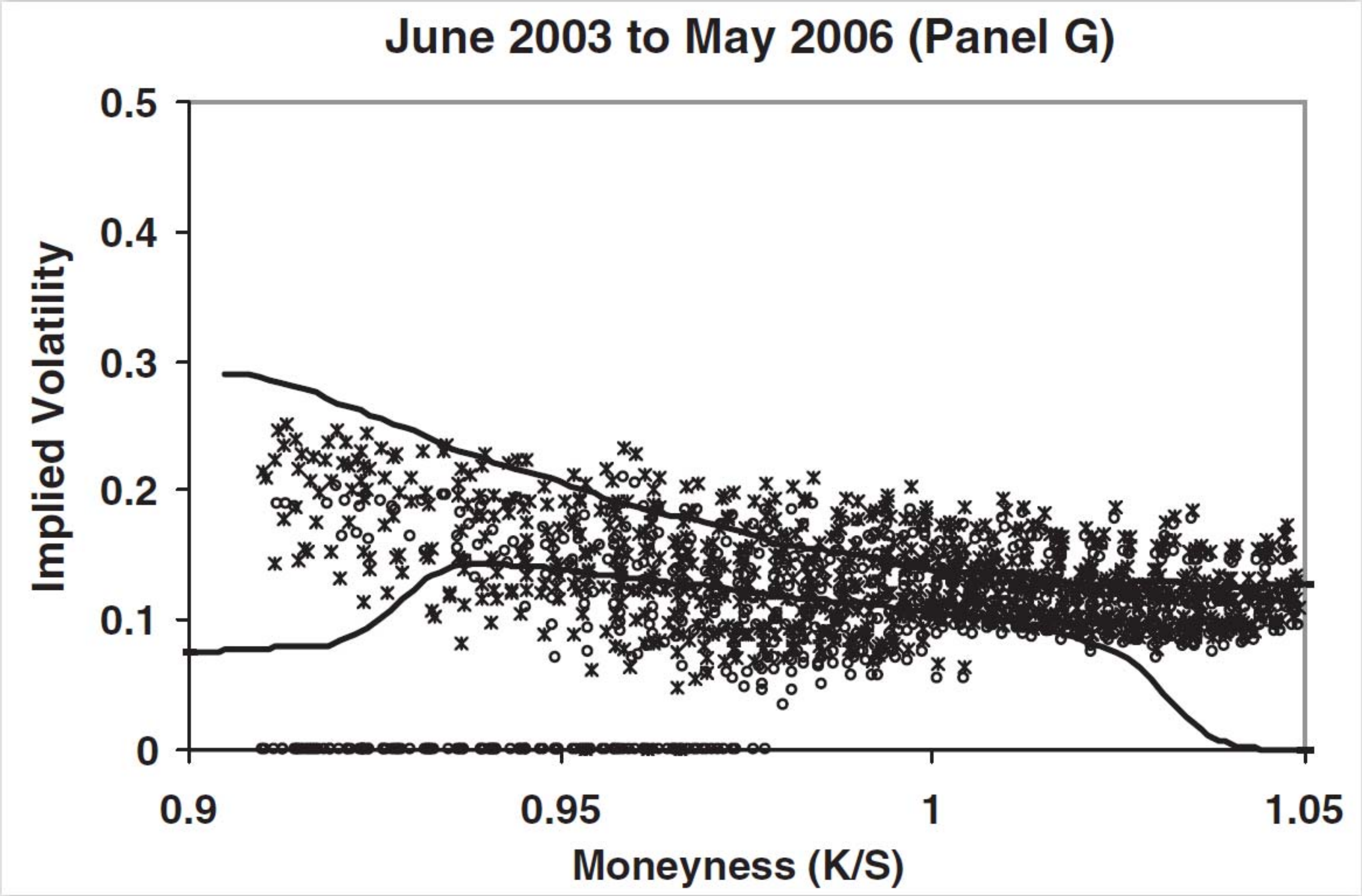
must be at least 20 for each quote. Our data also show that the average size of the violation is between 5% and 56% of the upper bound for most methods

CJP, Fig. 3



February 2000 to May 2003 (Panel F)
Implied Volatility
0.5
0.4
0.3
0.2
0.1
0
0.9
0.95
1
1.05
Moneyness (K/S)

CJP, Fig. 4



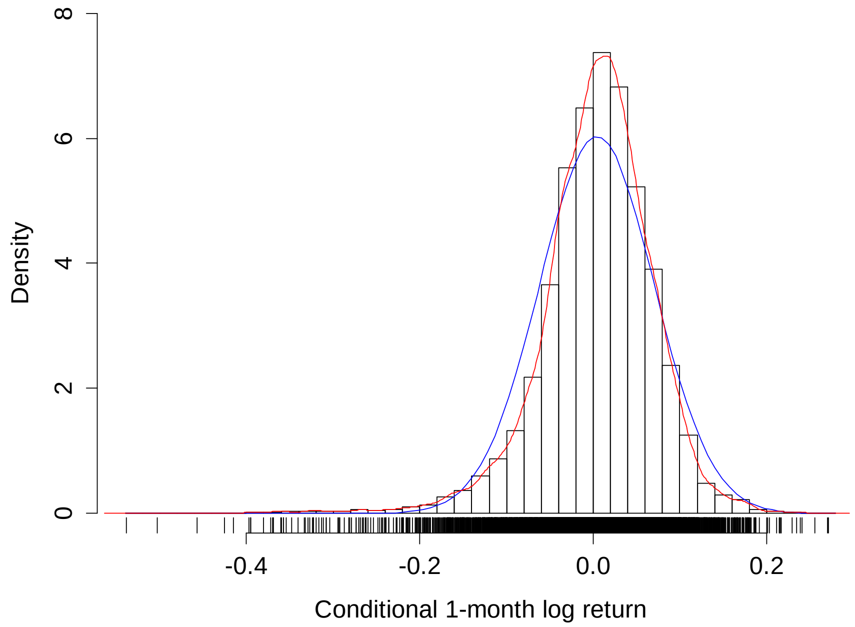
June 2003 to May 2006 (Panel G)
Implied Volatility
0.5
0.4
0.3
0.2
0.1
0
0.9
0.95
1
1.05
Moneyness (K/S)

Data

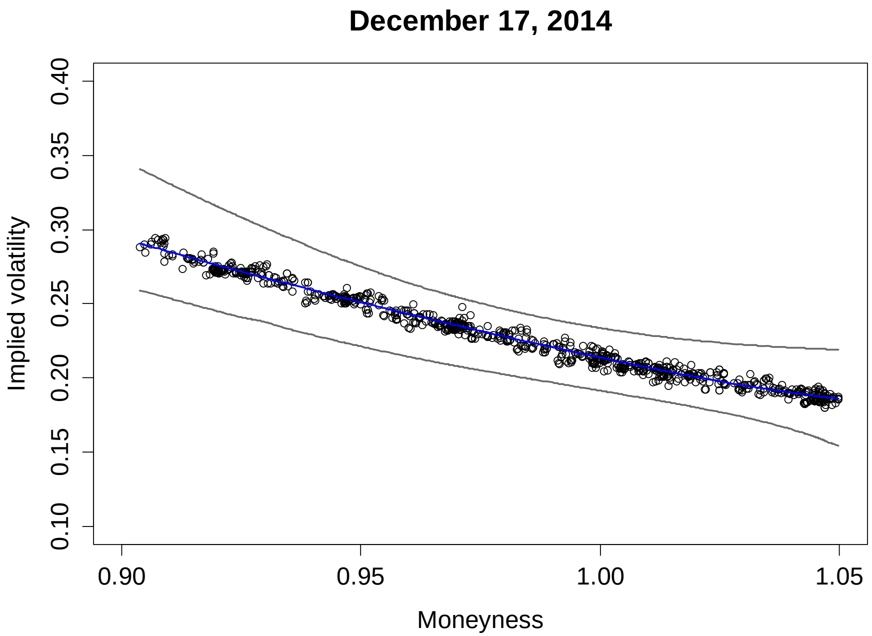
- DAX option: 1995 to 2014, ESX option: 2000 to 2014
- Transaction data from Eurex
- Time to maturity of 30 days (one such option per month)
- Matching of option and futures data by milliseconds
- Adjustment of futures prices (per option series) to ensure put-call parity
- Cubic regression model of implied volatility on moneyness explains, on average, approximately 97% of the variation in implied volatility

- Unconditional objective distribution: smoothed historical distribution of index returns over 1972-2006 (ESX: 1987-2006)
- Conditional volatility: such that the actual level of the smile is reproduced

Example



Density
0
2
4
6
8
-0.4
-0.2
0.0
0.2
Conditional 1-month log return

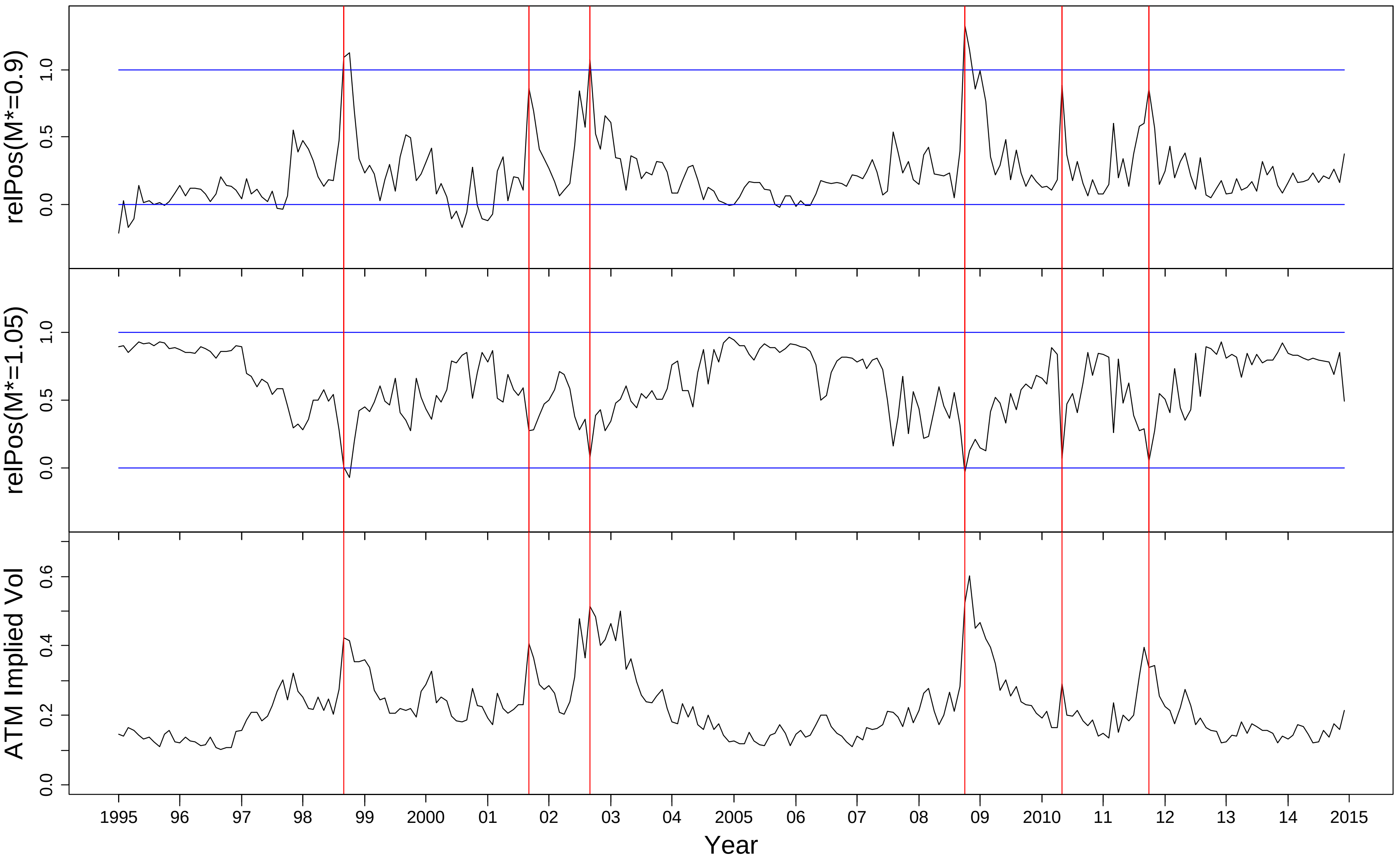


December 17, 2014
Implied volatility
0.10
0.15
0.20
0.25
0.30
0.35
0.40
0.90
0.95
1.00
1.05
Moneyness

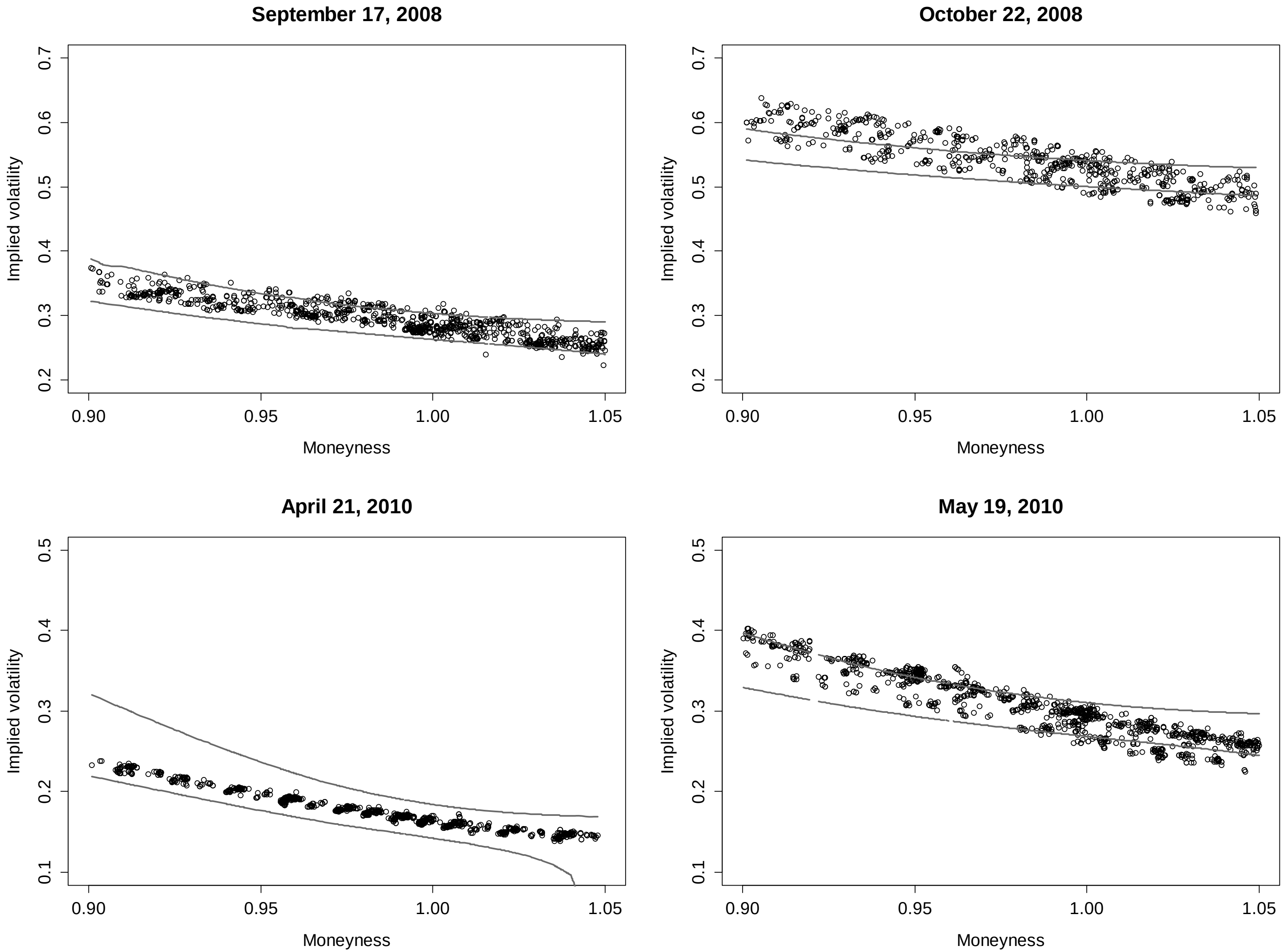
Results ESX 2000-2014

	Puts		Calls		All	
	Panel A: All transactions					
	N	in %	N	in %	N	in %
Upper violation	766	0.4	294	0.3	1'060	0.4
Inside bounds	173'080	96.6	108'200	99.4	281'280	97.6
Lower violation	5'414	3.0	311	0.3	5'725	2.0
Sum	179'260	100.0	108'805	100.0	288'065	100.0
	Mean	in %	Mean	in %	Mean	in %
Upper deviation IV	0.0066	1.7	0.0052	1.5	0.0062	1.6
Lower deviation IV	0.0028	1.2	0.0057	1.9	0.0030	1.2

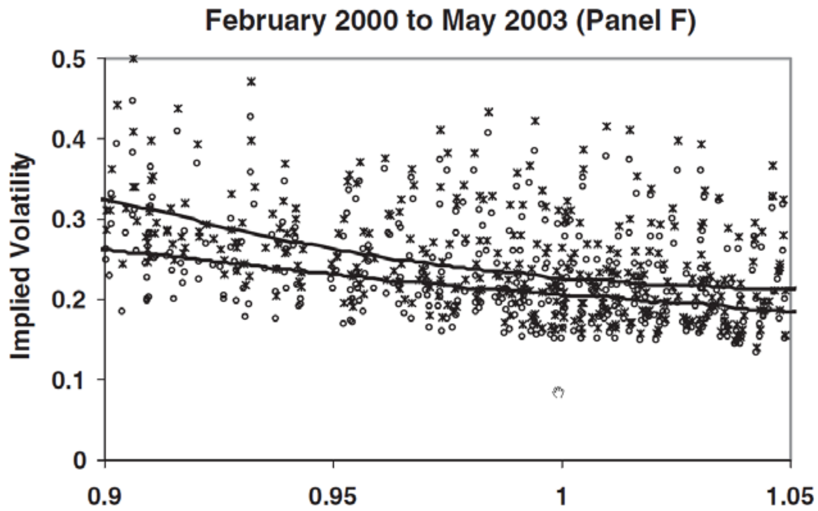
Timeline of violations for DAX

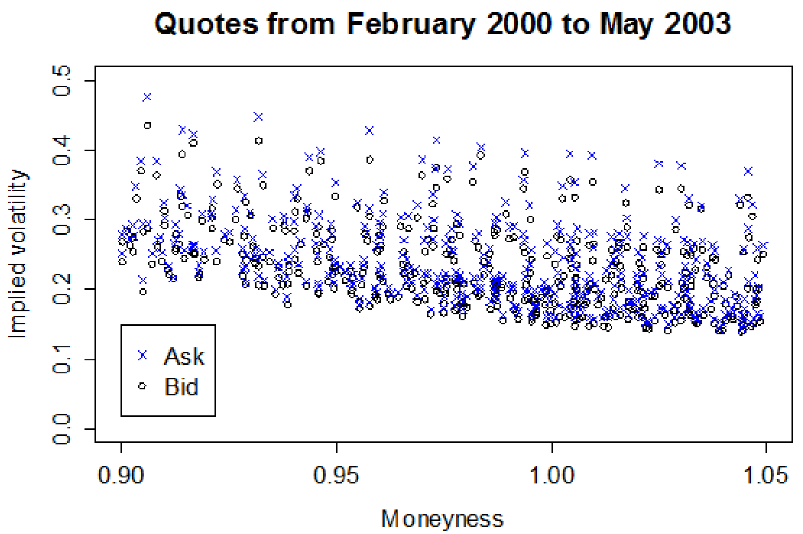


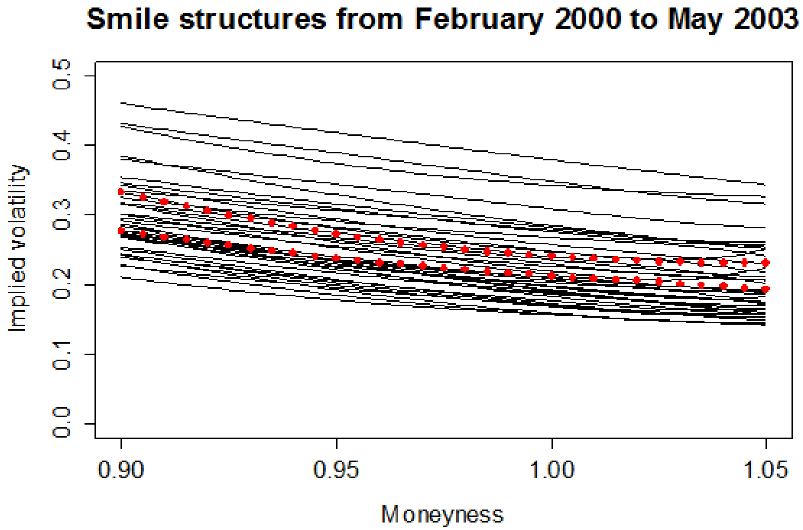
Crisis events



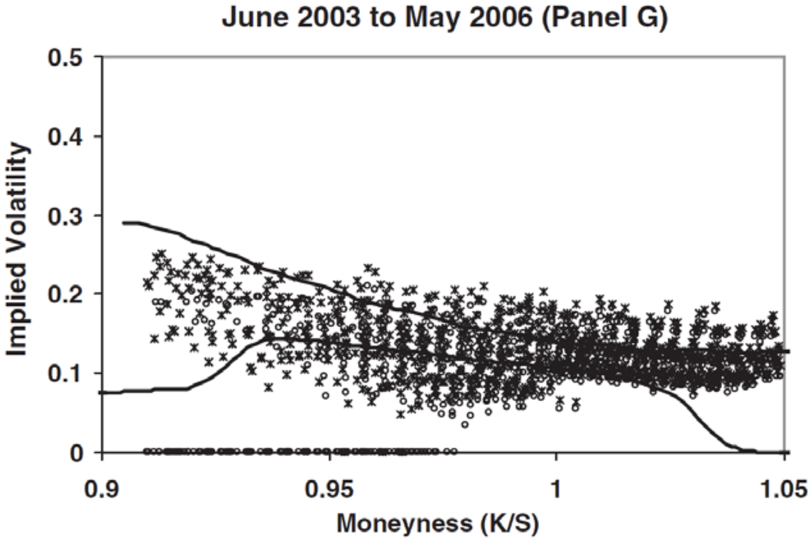
Replication of CJP (2009)

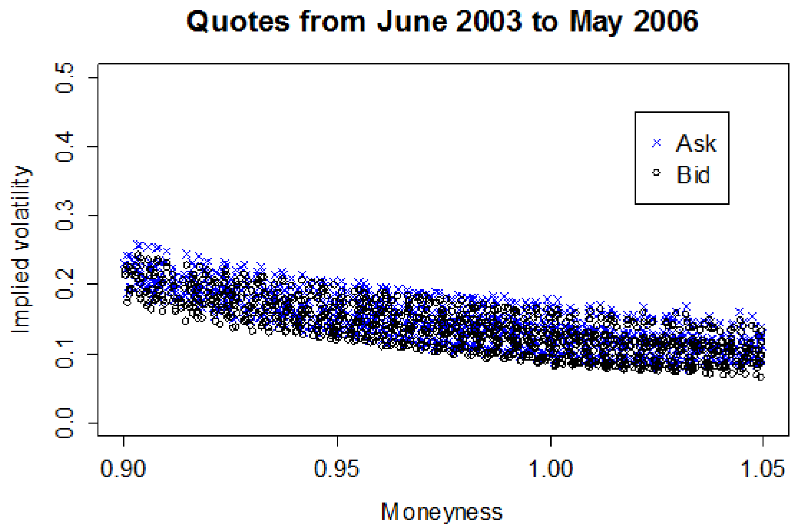


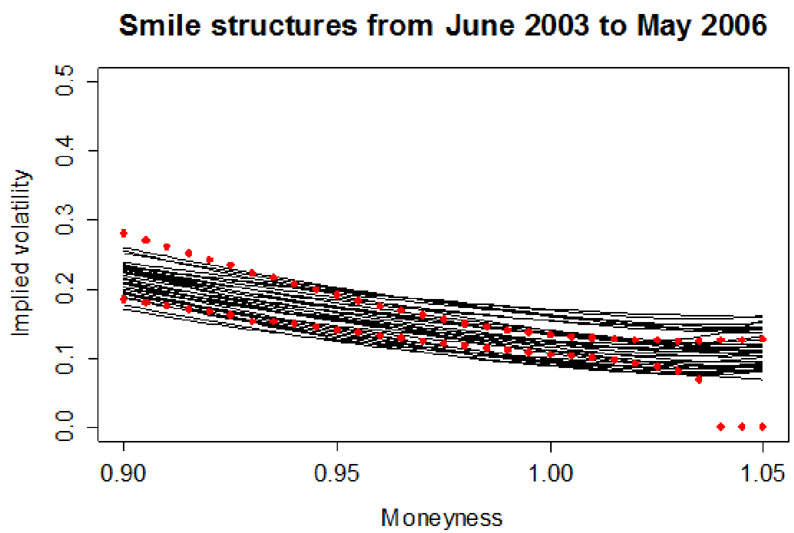




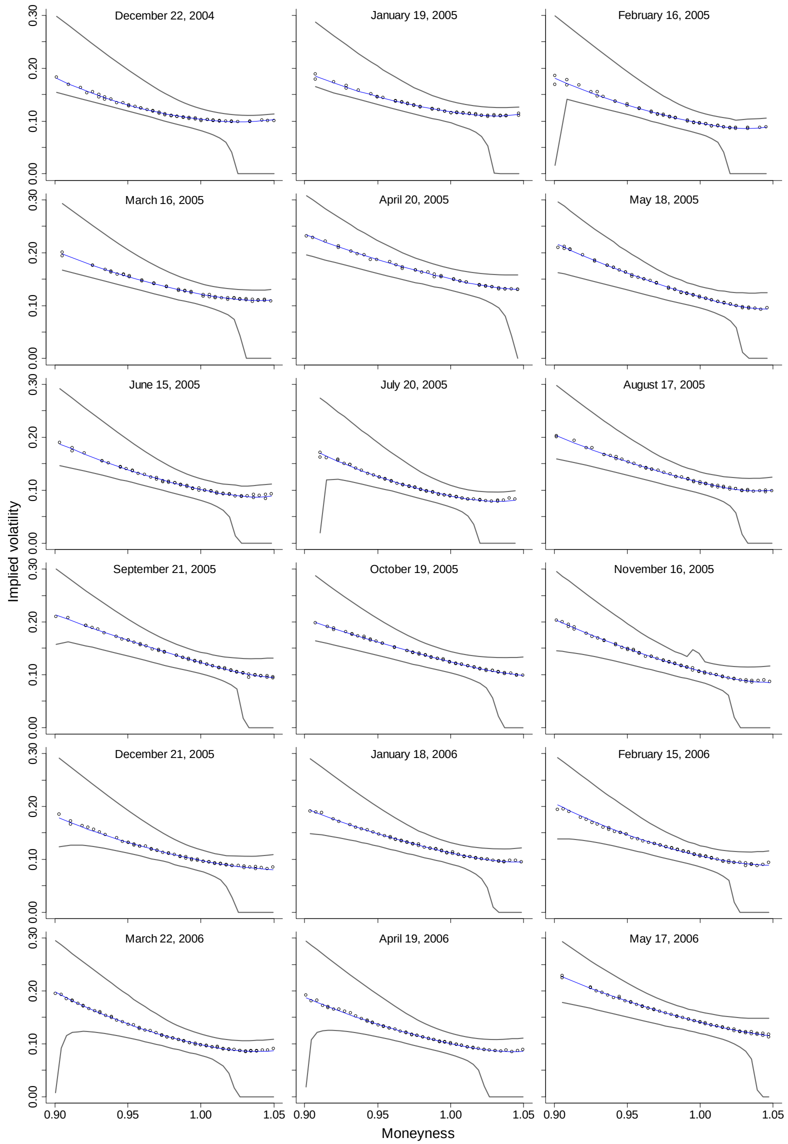
Replication ctd.

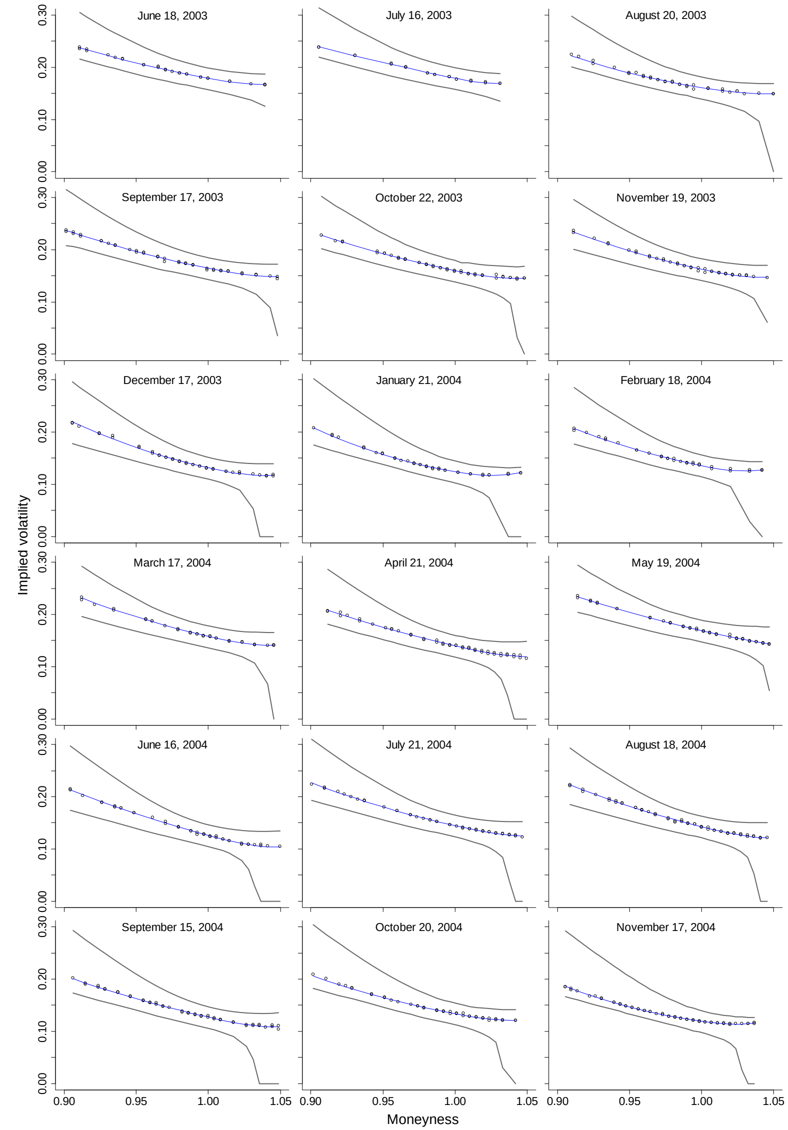






Smiles and bounds last subperiod (36 months)

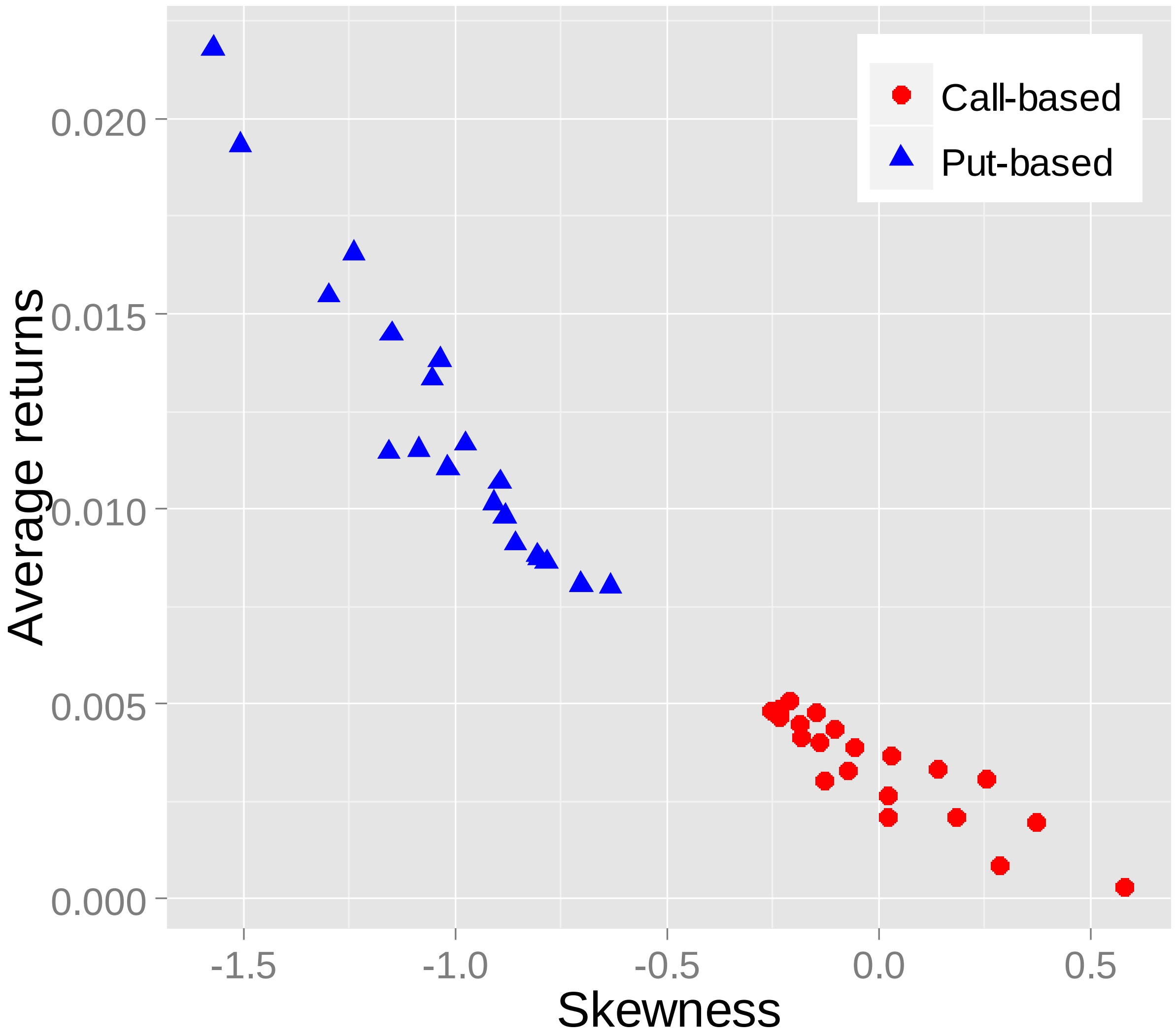




Skewness Premium and Index Option Returns

- Constantinides/Jackwerth/Savov (2013) propose a methodology to create a sample of option portfolios with an index exposure of one.
- Motivation:
  - Test linear factor models
  - Establish a new set of test portfolios for asset pricing tests (return data publicly available in online appendix)
- "The major advantage of this construction is to lower the variance and skewness of the monthly portfolio returns and render the returns close to normal (about as close to normal as the index return), thereby making applicable the standard linear factor pricing methodology." (p. 2)
- Main finding: cross-section of option returns is well explained by the sensitivity to any one of the risk factors price jumps, volatility jumps, and liquidity (along with the market).

Returns "about as close to normal as the index itself"?



Skewness premium: estimation approaches

- Skewness premium from a model-free strategy which creates a payoff equal to realized market skewness (see, e.g., Kozhan/Schneider/Neuberger, 2013)
- Skewness as a priced factor in the cross-section of stocks
- Market skewness as a risk factor.

  Test if expected stock returns are related to the sensitivity of stocks with respect to changes in market skewness.

  Chang/Christoffersen/Jacobs (2013):

  “We find that the average return on the market skewness risk factor portfolio is 0.78% per month, or 9.36% per year, and this return cannot be explained by market beta, the size factor, the book-to-market factor, or the momentum factor.”

Portfolio construction

- Deleverage options so that their returns have an elasticity to index returns of 1
- Proportion of wealth invested in the option:

$$\omega = \frac{1}{\frac{\partial \mathcal{C}}{\partial S}\frac{S}{\mathcal{C}}}$$

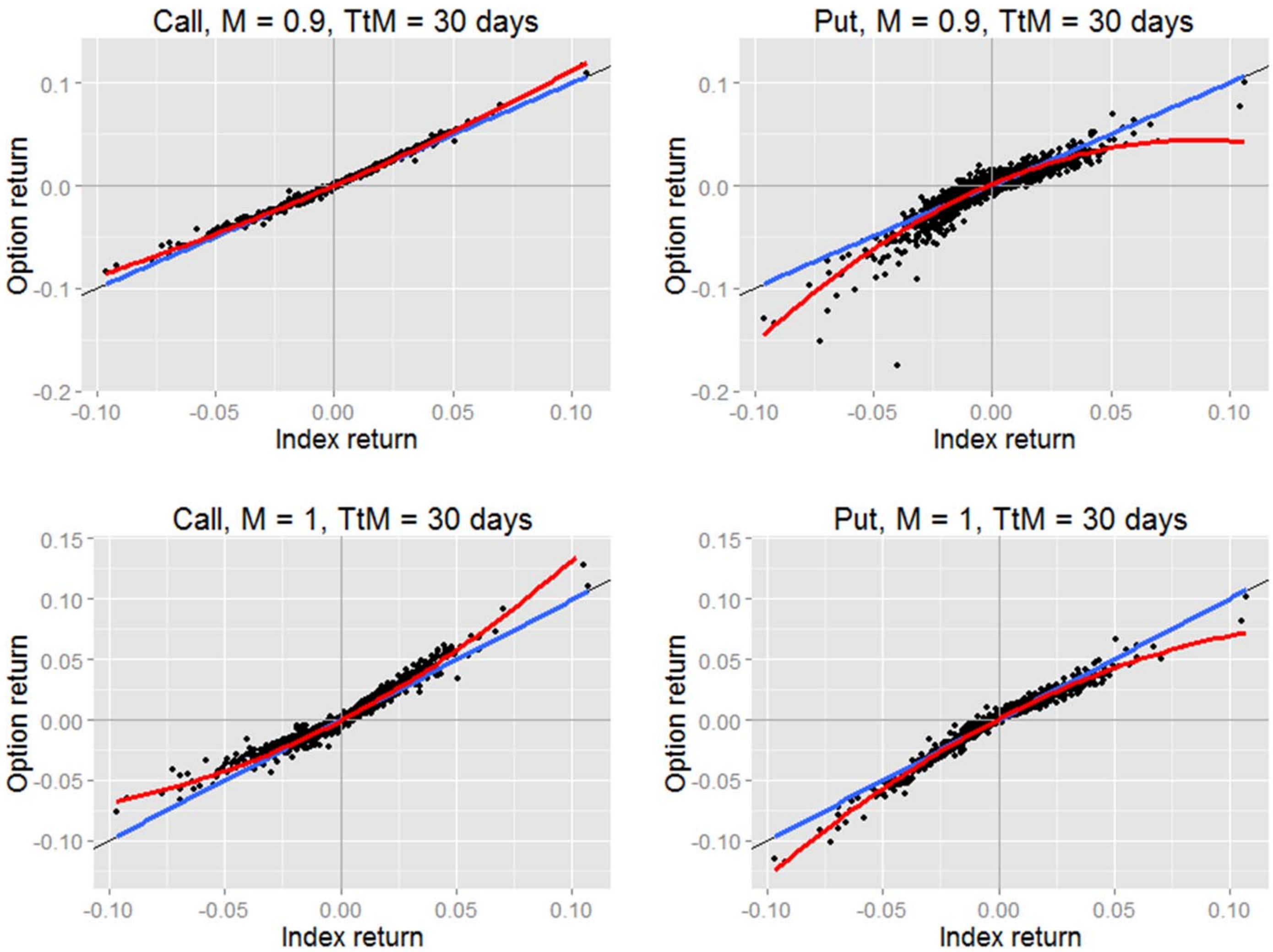
Proportion $(1-\omega)$ invested in the riskless asset.

$$\frac{d\mathcal{C}(S,\sigma_{imp}(S))}{dS} = \frac{\partial \mathcal{C}(S,\sigma_{imp}(S))}{\partial S} + \frac{\partial \mathcal{C}(S,\sigma_{imp}(S))}{\partial \sigma_{imp}} \cdot \frac{d\sigma_{imp}(S)}{dS}$$

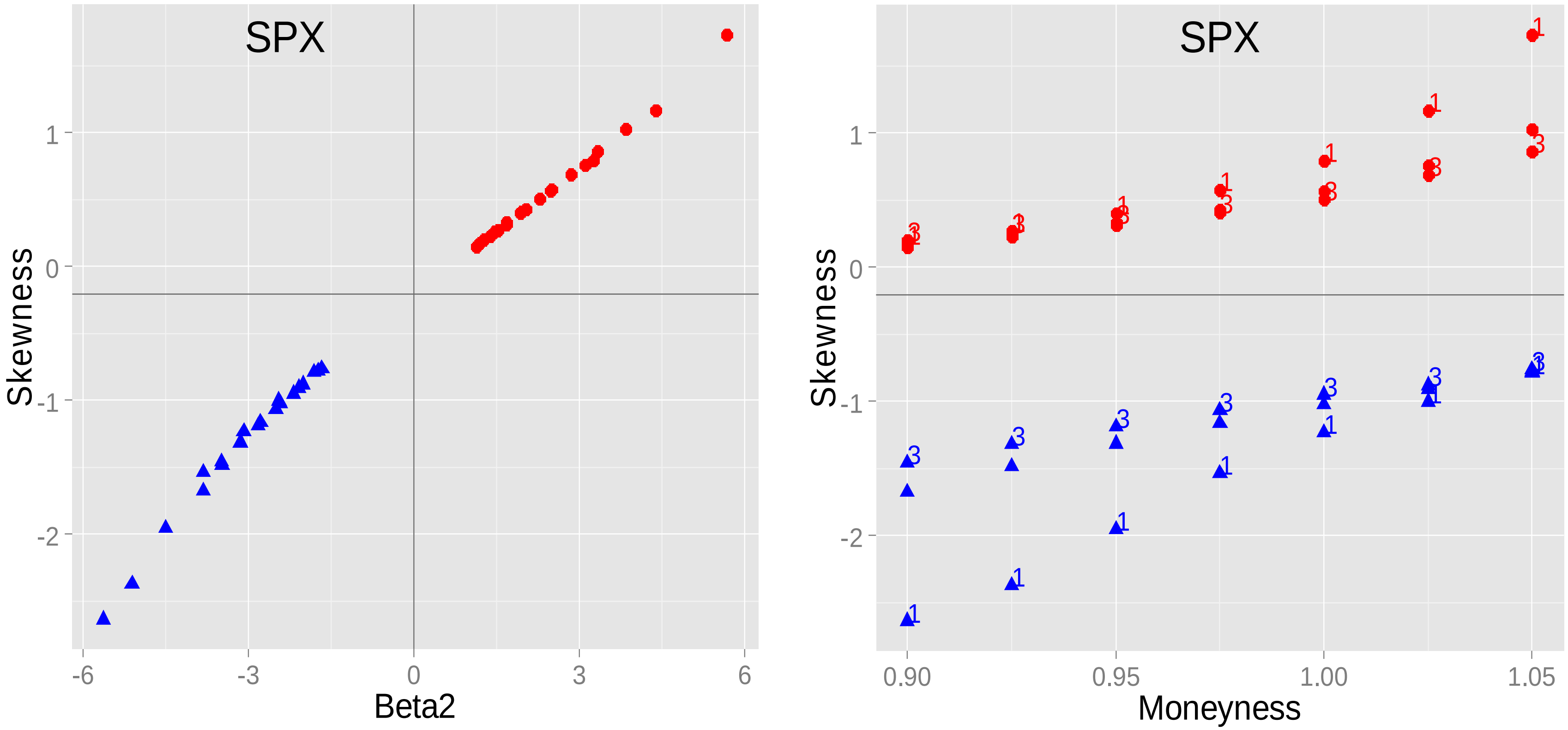
$$= \Delta_{BS} + \Lambda_{BS} \cdot \frac{d\sigma_{imp}(S)}{dS}$$

- Sequential 1-day investments.
- $T \in \{30, 60, 90\}$; $m \in \{0.9, 0.925, 0.95. 0.975, 1.0, 1.025, 1.05\}$; $z \in \{call, put\}$

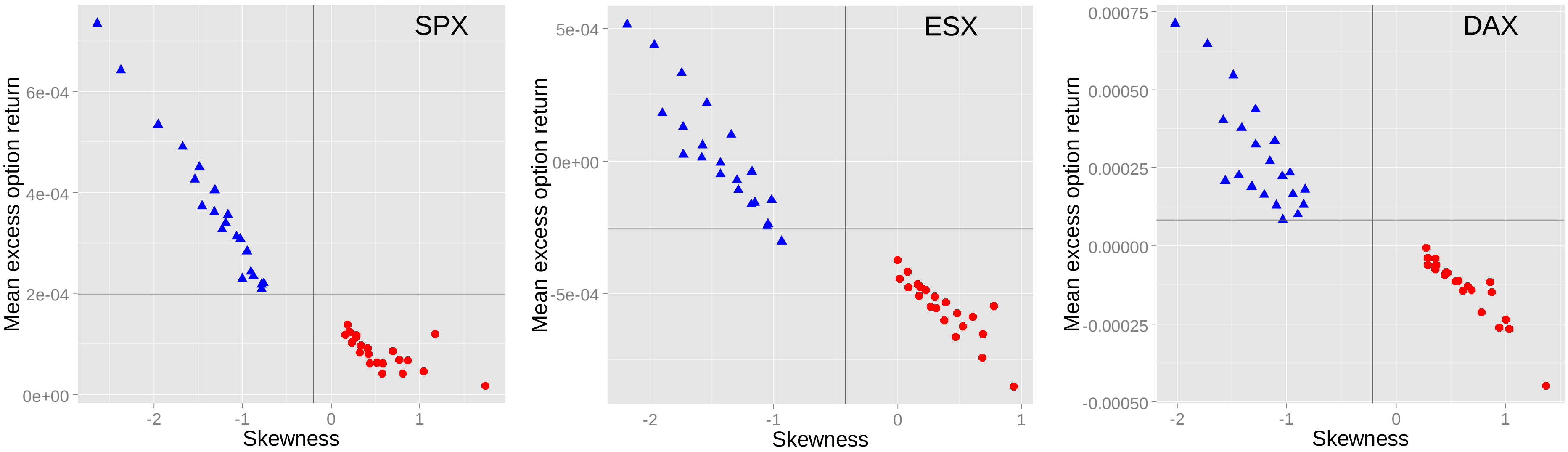
SPX, Option Metrics data, 1996-2015



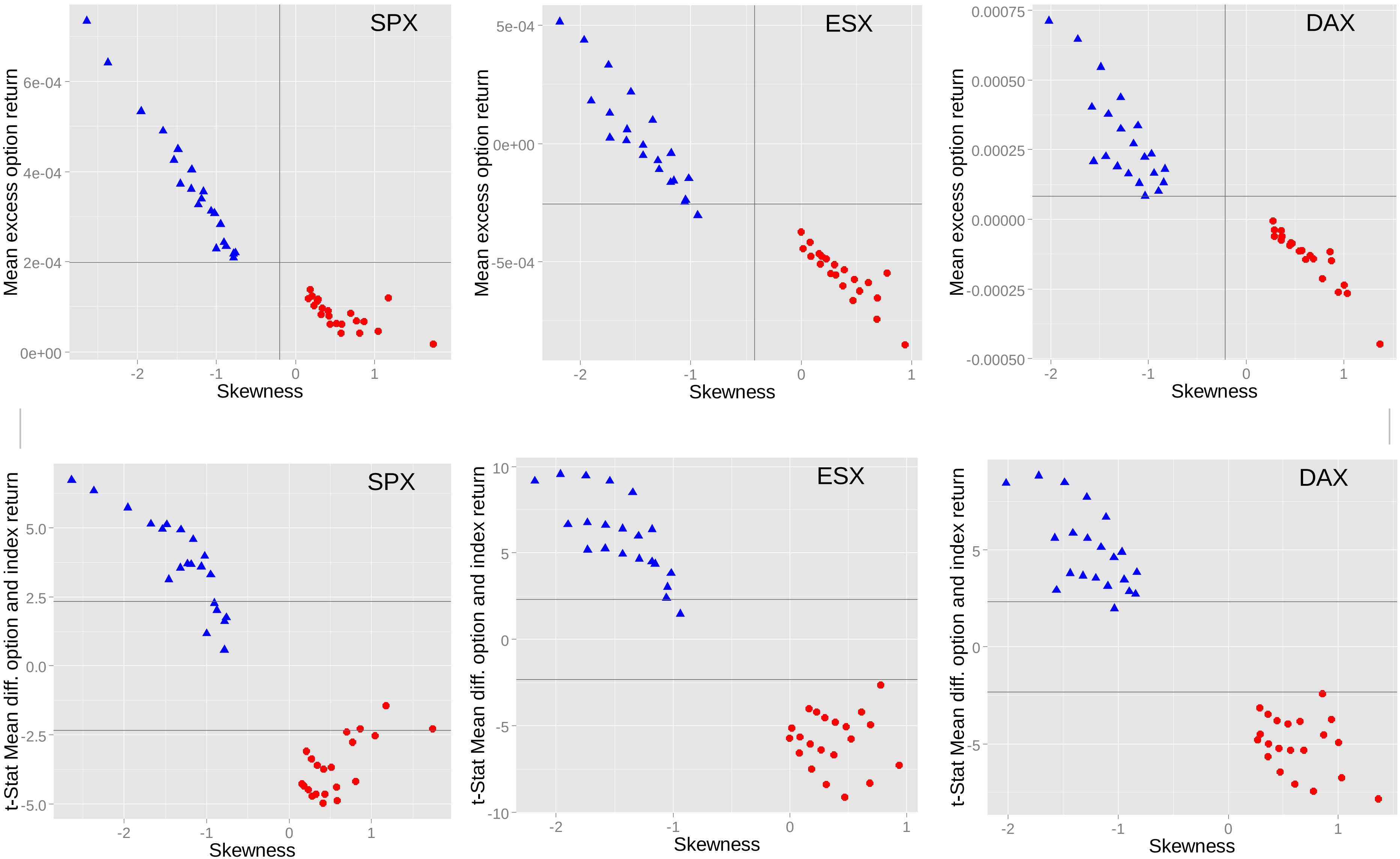
Skewness and quadratic index sensitivity



Skewness and average returns



Skewness and average returns



Skewness premium

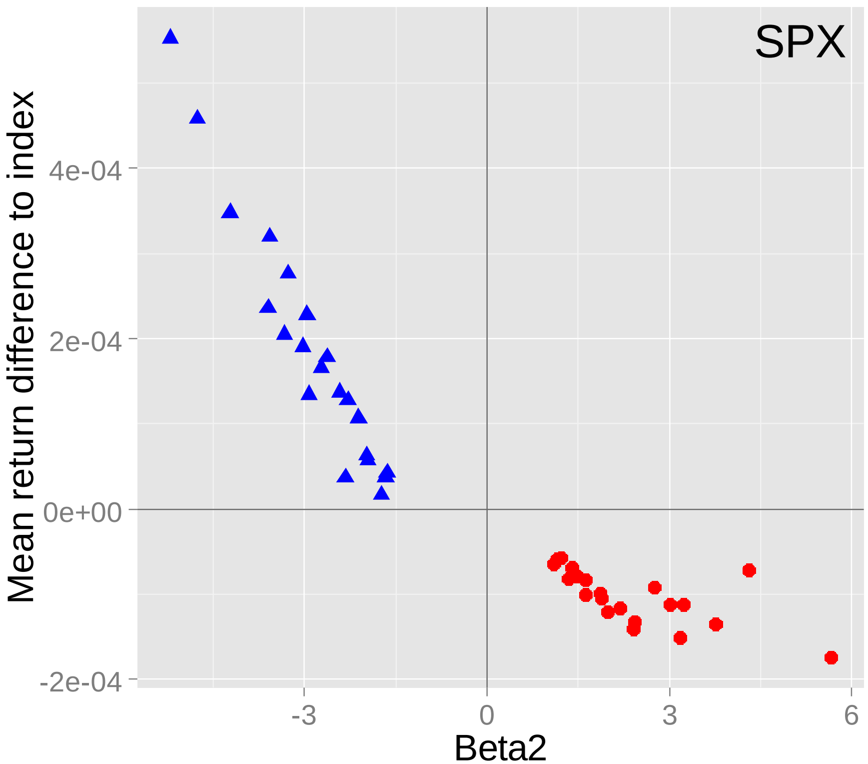
$$\text{Model 1:} \quad r_{pt} = \alpha + \beta x_{s,p} + \varepsilon_{pt}$$

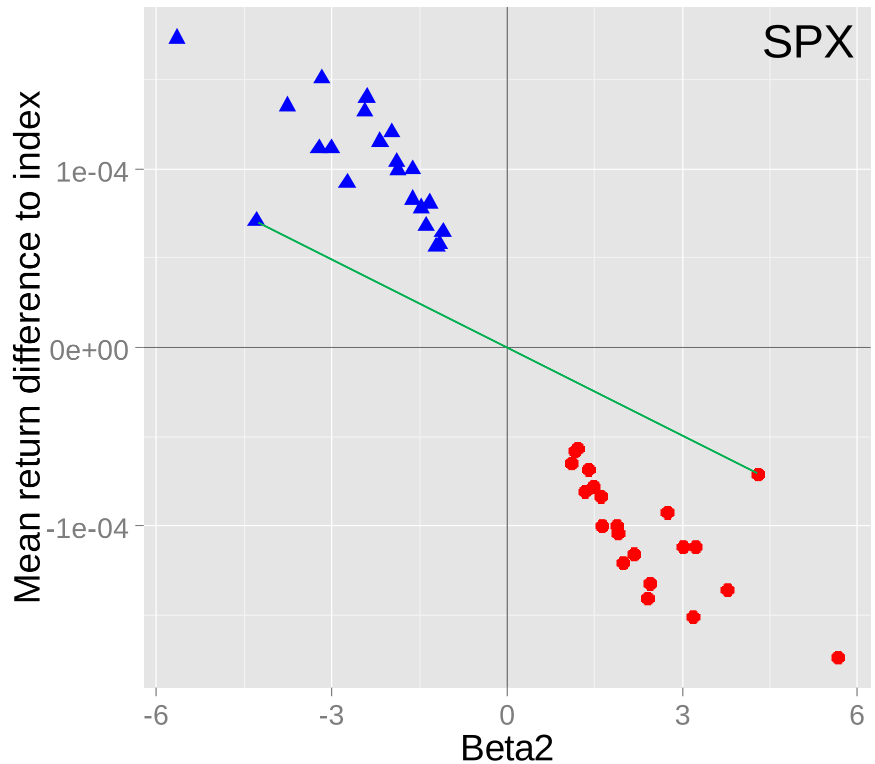
$$\text{Model 2:} \quad r_{pt} = \alpha + \beta x_{s,p} + \gamma x_{s,p}^2 + \varepsilon_{pt} \qquad x_s \in \{Skew, Beta2\}$$

$$\text{Model 3:} \quad r_{pt} = \alpha + \beta x_{s,p} + \gamma k_p + \varepsilon_{pt}$$

	Model 1			Model 2				Model 3			
Panel B: Excess option returns to index, excess skewness and kurtosis compared to index											
	Interc	Skew	R^2	Interc	Skew	Skew^2	R^2	Interc	Skew	Kurt	R^2
SPX	1.24E-04	-1.58E-04	0.905	8.27E-05	-1.38E-04	4.07E-05	0.973	8.92E-05	-1.09E-04	2.06E-05	0.971
	1.19	-14.25		-2.87	-11.46	3.47		-2.48	-6.36	3.42	
ESX	7.88E-05	-3.36E-04	0.934	3.17E-05	-3.16E-04	5.41E-05	0.943	3.44E-05	-2.47E-04	2.39E-05	0.944
	-0.78	-24.87		-3.11	-20.71	2.34		-3.35	-6.59	2.46	
DAX	1.04E-04	-2.42E-04	0.901	7.45E-05	-2.36E-04	3.09E-05	0.907	6.33E-05	-2.50E-04	1.73E-05	0.914
	0.56	-20.64		-1.47	-20.00	1.57		-2.16	-19.37	2.21	
	Interc	Beta2	R2	Interc	Beta2	Beta2^2	R2	Interc	Beta2	Kurt	R2
SPX	1.34E-04	-5.41E-05	0.855	6.97E-05	-5.06E-05	7.45E-06	0.958	8.77E-05	-3.49E-05	2.47E-05	0.970
	2.30	-14.29		-3.61	-13.20	4.41		-2.64	-6.53	4.45	
ESX	9.37E-05	-1.01E-04	0.933	3.42E-05	-9.56E-05	5.99E-06	0.945	3.99E-05	-7.12E-05	2.65E-05	0.946
	0.62	-25.21		-2.84	-22.59	2.79		-2.91	-6.78	2.87	
DAX	9.89E-05	-7.01E-05	0.882	6.40E-05	-6.80E-05	3.14E-06	0.888	4.52E-05	-7.34E-05	2.26E-05	0.902
	0.01	-20.81		-1.93	-19.47	1.65		-3.18	-19.46	2.82	

Put-call parity





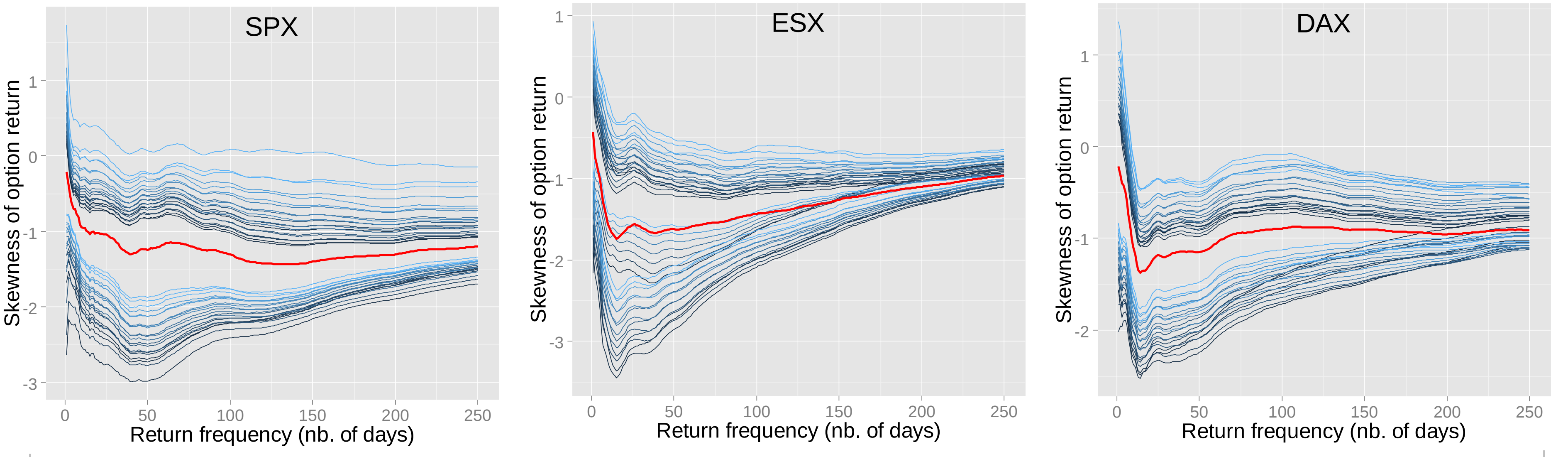
All combinations of Index with Beta-One-Option lie on a straight line through the origin.

Denote by *w* the weight of the Beta-One-Option so that (1-*w*) is the Index weight.

Put and call based daily returns are equal if $d_p/w_p = d_c/w_c$

where d_p and d_c are the option deltas. $w_c = 1 \Rightarrow w_p = d_p/d_c = 1 - 1/d_c$

Skewness and investment horizon



Conclusion

- The method proposed by Constantinides et al. (2013) can be used to estimate the market skewness premium in a new and simple way.

  → interesting representation of the smile and its dynamics

- The main finding is that the pricing of index options is consistent with a substantial negative skewness premium.

- An overall preference for positive skewness could explain why the risk-neutral distribution implied in option prices is so strongly skewed and could also contribute to resolving the pricing kernel puzzle.

- Of course, skewness is closely related to the risk of negative index returns (e.g. Seo, 2015, with further references). In this sense, results are in line with the findings of CJS.