Hedge Fund Portfolio Management with Illiquid Assets

Serge Darolles[1], Guillaume Roussellet[2]

[1] *Université Paris-Dauphine and* Crest

[2] Nyu *Stern School of Business - Volatility Institute*

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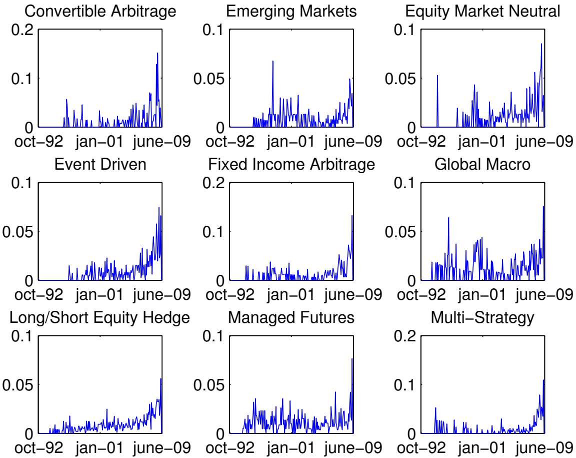
Contents

Hedge funds liquidation rates

- The majority of Hedge funds have rather short lifetimes (compared to mutual funds)
- The liquidation rates, as well as the dependence among them, vary considerably according to:
  - The period of time
  - The management style

Frequency Counts of Liquidated Hedge funds



What are the drivers of Hedge fund liquidations ?

- Individual drivers
  - Illegal operations or frauds
  - Insufficient incentives for the Hedge fund manager
  - Poor performance: the Hedge fund is not attractive for investors

- **Common drivers**
  - **Liquidity Crises**

- How can we use liquidation date to infer fund exposures to liquidity risk ?

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With Hedge funds data ...

- Large sample of different Hedge fund strategies ...
- ... with different exposures to market liquidity (market portfolios)
- ... with different exposures to funding liquidity (leverage, redemption policies)
- And then different liquidity mismatchs !

Hedge funds and exposures to liquidity

i) Examples of funding liquidity shocks

- Large cash withdrawals of direct investors during **funding liquidity crisis**
- Deleveraging imposed by prime brokers
- Outflow of funds of funds investors

Effect by means of the **liability component of the balance sheet**

Hedge funds and exposures to liquidity

ii) Example of market liquidity shocks

Fund 1 is liquidated and sells illiquid asset A

→ “market price" of asset A decreases

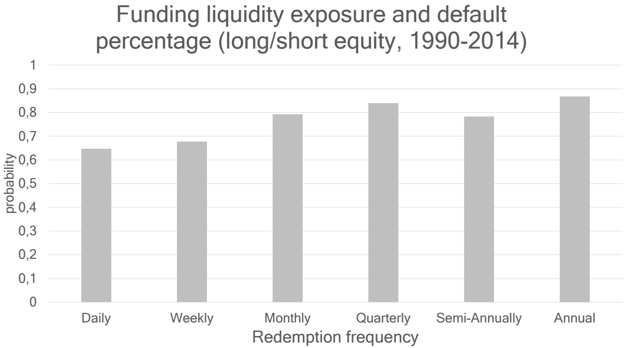
→ NAV of **Fund 2** invested in asset A decreases

→ likelihood of selling asset A by of **Fund 2** increases

Effect by means of the **asset component of the balance sheet**

Hedge funds and exposures to liquidity

- High level of *liquidity transformation*
- However, great resilience to depressed market liquidity and fire-sales during the *Financial Crisis* ...



Previous studies (with return data ....)

- Previous empirical studies have shown that both market **and** funding liquidity are priced in the cross-section of fund's returns.

Market: Sadka 2006-2010, Teo 2011.
Funding: Dudley & Nimalendran 2010, Aragon & Strahan 2012.
Both: Agarwal, Aragon & Shi 2015.

- They mostly use regression evidence using market and funding liquidity indicators on the right-hand side.
- Hombert & Thesmar 2014 endogenize the funding liquidity exposure via constraints on withdrawals.

Endogeneity issue

The fund manager adjusts his portfolio [cash holdings] assessing future market and funding liquidity conditions.

In this paper

- We propose a two-period structural model of a single hedge fund portfolio management when it is subject to both funding and market liquidity shocks. [closest to ours is Liu & Mello 2011]
- The funding liquidity shock is a random quantity of AUM being withdrawn, whereas the market liquidity shock is a random haircut applied on the fund's risky assets.
- The model produces closed-form solution for the fund's optimal cash holdings as a function of market and funding liquidity conditions, allowing to derive testable implications on hedge fund's data.

Results

- Our model produces endogenous default probabilities, default arising as the conjunction of big funding and market liquidity shocks together.
- The fund's optimal cash amount allows it to hedge partly against bad funding and market liquidity conditions (Liquidity timing).
- The fund's optimal cash amount allows it to mitigate the effect of market liquidity shocks when the market liquidity is very high and very low (Liquidity resilience).
- We confirm previous results showing that the fund's value is increasing in the funding liquidity conditions (Mismatch reduction).
- Taking the model to the data is easy and produces meaningful results with a new mismatch factor(*ongoing*).

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Fundamental assumptions

Consider a single hedge fund entity living for 2 periods:

- risk-neutral,
- price-taker,
- has a unit amount of assets under management,

which has access to two assets:

- **Cash:** zero interest rate, available every period;
- **Illiquid asset:** positive deterministic interest rate $\rho_1 + \rho_2$, available at period $t = 2$ only.

The HF balance-sheet is:

Assets	Liabilities
Cash Illiquid asset	AUM

Timing

$t = 0$ $\rightarrow$ The fund chooses a portfolio composition $(\delta, 1-\delta)$ in cash/illiquid asset.

$t = 1$ $\rightarrow$ With probability π, a fraction θ of AUM is withdrawn by the investors.

- $\theta \sim \mathcal{U}[0, \overline{\theta}]$, where $\overline{\theta} \leqslant 1$.
- If $\theta \leqslant \delta$, the fund takes its cash to pay the investors.
- If $\theta > \delta$, the fund has to sell the illiquid asset on a secondary market.

$t = 2$ $\rightarrow$ The final value of the fund's portfolio is realized.

Funding liquidity shock

The value of θ is the funding liquidity shock. It is associated with parameters π and $\overline{\theta}$.

Liquidation cost on the secondary market

Remember that the illiquid asset has a known return $\rho_1 + \rho_2$ at period $t = 2$.

- ρ_1 alone would be the friction-less return at period $t = 1$.

Market liquidity shock

The realized return of the illiquid asset on the secondary market is equal to $\rho_1 - T$, where T is the market liquidity shock.

- $T \sim \mathcal{E}(\lambda)$, where $\lambda > 0$.
- λ is an indicator of market liquidity, $1/\lambda$ is the average rebate on the secondary market.

When does default arise?

Two situations can arise when a "big" θ is observed:

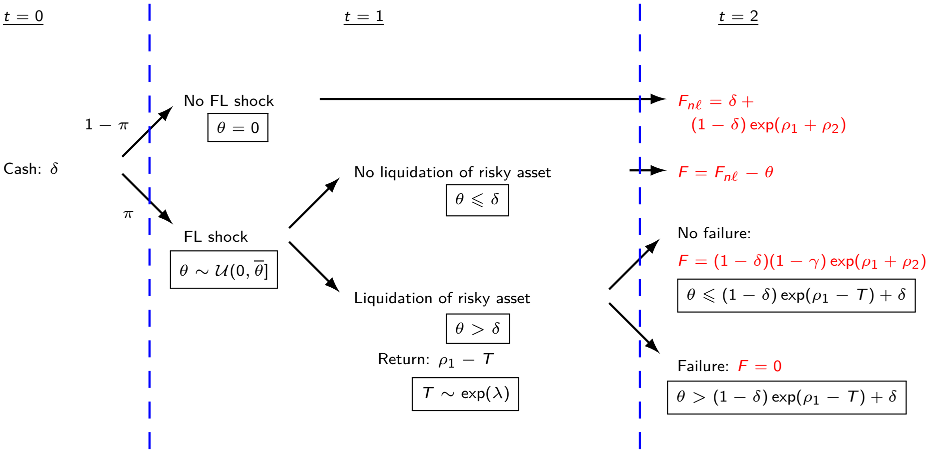
- The realized return $\rho_1 - T$ is sufficiently high to cover the discrepancy $\theta - \delta$. I sell a proportion γ of the illiquid asset equal to:

$$\boxed{\gamma(1-\delta)\exp(\rho_1 - T) = \theta - \delta \Longleftrightarrow \gamma = \frac{\theta - \delta}{1-\delta}\exp(T - \rho_1)}$$

- If T is too high, selling all the illiquid asset is not sufficient to cover $\theta - \delta$. The default is triggered whenever:

$$\boxed{(1-\delta)\exp(\rho_1 - T) < \theta - \delta \Longleftrightarrow \gamma > 1}$$

Summary of the setup



The fund's maximization problem

- The fund is looking for the optimal quantity δ to maximize its expected portfolio value $\mathbb{E}(F)$ at period $t = 2$ from $t = 0$ perspective.

$$\delta^* = \underset{\delta}{\operatorname{argmax}}\, \mathbb{E}[F(\delta, \overline{\theta}, \lambda, \pi, \rho_1, \rho_2)] \quad \text{s.t.} \quad \delta \in [0, 1]\,.$$

- The expected portfolio value is decomposed with the law of iterated expectations and needs the conditional distribution of γ given a big funding liquidity shock.

Default probability with exogenous cash

- Using the properties of the Uniform and the Exponential distributions, it is possible to obtain the conditional distribution of γ given a big liquidity shock.
- The unconditional default probability is a by-product:

Default probability

$$\mathbb{P}(\gamma > 1) = \frac{\pi(\overline{\theta} - \delta)^{\lambda+1}}{\overline{\theta}(\lambda + 1)(1 - \delta)^{\lambda}} e^{-\lambda \rho_1}$$

- This probability is decreasing in the market liquidity λ, increasing in the size of the maximal possible funding shock $\overline{\theta}$ and decreasing in the cash amount δ.

The expected fund's portfolio value

- The expected value of the fund's portfolio from $t = 0$ perspective is available in closed-form:

$$
\begin{aligned}
\mathbb{E}(F) &= \overbrace{(1-\pi)\left[\delta + (1-\delta)\exp(\rho_1+\rho_2)\right]}^{\text{no liquidity shock}} + \overbrace{\pi\frac{\delta}{\overline{\theta}}\left((1-\delta)\exp(\rho_1+\rho_2) + \frac{\delta}{2}\right)}^{\text{small liquidity shock}} \\
&+ (1-\delta)\exp(\rho_1+\rho_2)\pi\frac{\overline{\theta}-\delta}{\overline{\theta}} \underbrace{\left[1 - \frac{1}{\lambda+1}\left(\frac{\overline{\theta}-\delta}{(1-\delta)\exp(\rho_1)}\right)^{\lambda}\right]}_{\text{conditional survival probability}} \\
&\times \underbrace{\left\{1 - \frac{\lambda}{1-\lambda^2}\frac{\overline{\theta}-\delta}{(1-\delta)\exp(\rho_1)}\left[\left(\frac{\overline{\theta}-\delta}{(1-\delta)\exp(\rho_1)}\right)^{\lambda-1} - \frac{\lambda+1}{2}\right]\right\}}_{\text{conditional expectation of } 1-\gamma}.
\end{aligned}
$$

A specific case: $\overline{\theta} = 1$.

When $\overline{\theta} = 1$, the expectation is simplified:

$$\begin{aligned}\mathbb{E}(F) &= (1-\pi)\left[e^{\rho_1+\rho_2} + \delta(1-e^{\rho_1+\rho_2})\right] + \pi\left(\delta e^{\rho_1+\rho_2} + \delta^2\left(\frac{1}{2} - e^{\rho_1+\rho_2}\right)\right) \\ &+ (1-\delta)^2 e^{\rho_1+\rho_2}\pi\left[1 - \frac{e^{-\lambda\rho_1}}{\lambda+1}\right]\left\{1 - \frac{\lambda e^{-\lambda\rho_1}}{1-\lambda^2} + \frac{\lambda e^{-\rho_1}}{2(1-\lambda)}\right\}.\end{aligned}$$

which is **quadratic in** δ.

Model solutions

- When $\overline{\theta} = 1$, the problem can be solved in closed-form.
- When $\overline{\theta} < 1$, the problem has to be solved numerically.

Model solution when $\overline{\theta} = 1$

Optimal cash amount

$$\delta^*(1, \lambda, \pi, \rho_1, \rho_2) = \frac{G(\lambda, \rho_1) + H_1(\pi, \rho_1, \rho_2)}{G(\lambda, \rho_1) + H_2(\rho_1, \rho_2)} \quad \text{if this ratio is in } (0, 1)\,,$$

with

$$\begin{aligned} G(\lambda, \rho_1) &= \left[1 - \frac{e^{-\lambda\rho_1}}{\lambda + 1}\right]\left[1 - \frac{\lambda e^{-\lambda\rho_1}}{1 - \lambda^2} + \frac{\lambda e^{-\rho_1}}{2(1-\lambda)}\right] \\ H_1(\pi, \rho_1, \rho_2) &= \left(\frac{1}{\pi} - 1\right)\frac{1 - e^{-\rho_1 - \rho_2}}{2} - \frac{1}{2}, \quad H_2(\rho_1, \rho_2) = \frac{e^{-\rho_1 - \rho_2}}{2} - 1\,. \end{aligned}$$

- δ^* is a function of λ through $G(\cdot)$ only.
- δ^* is a function of π through $H_1(\cdot)$ only.

$\Longrightarrow$ this makes the influence of each parameter easier to calculate.

Properties of the optimal cash holding

The following properties hold when $\overline{\theta} = 1$:

Important results (1/3): liquidity timing

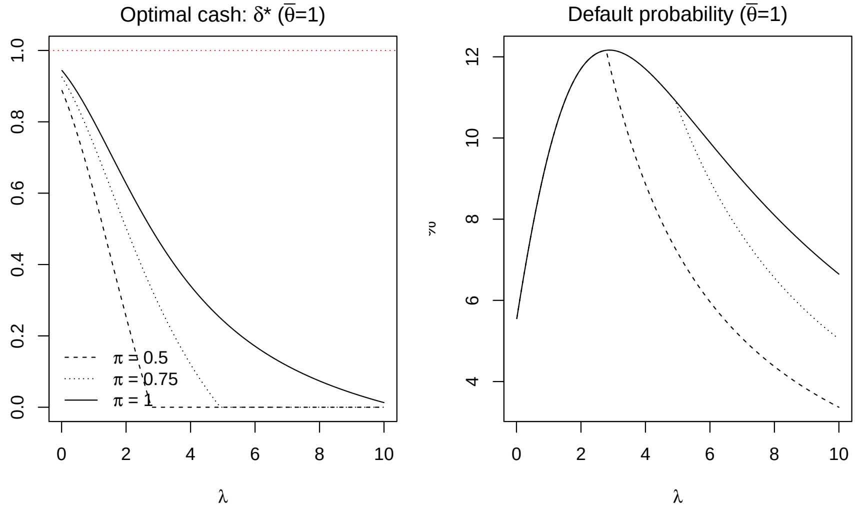
- δ^* is increasing with the probability of a funding liquidity shock π.
- δ^* is decreasing with the degree of market liquidity λ.

This allows us to derive the default probability when cash holdings are at the optimal value:

Important results (2/3): liquidity resilience

- The PD is insensitive to the probability of a funding liquidity shock π.
- The PD is a bell-shaped function of the degree of market liquidity λ.

Graphical illustration

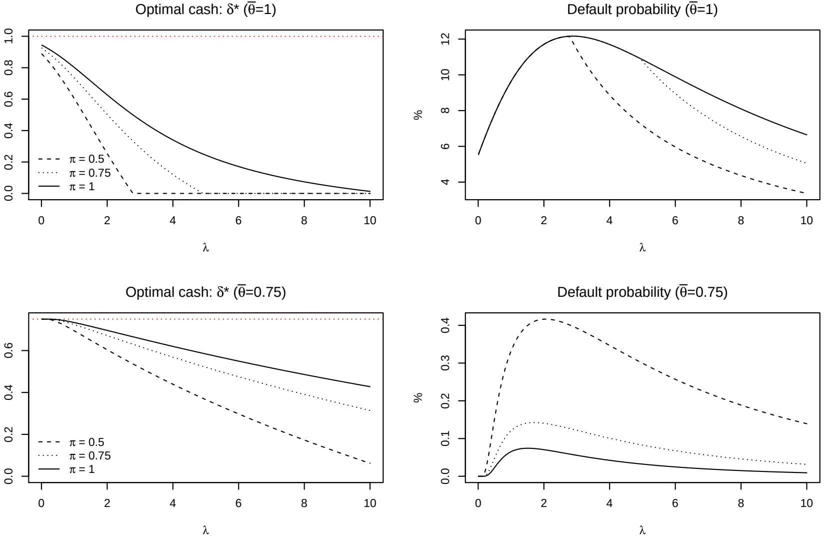


Model implications when $\overline{\theta} < 1$

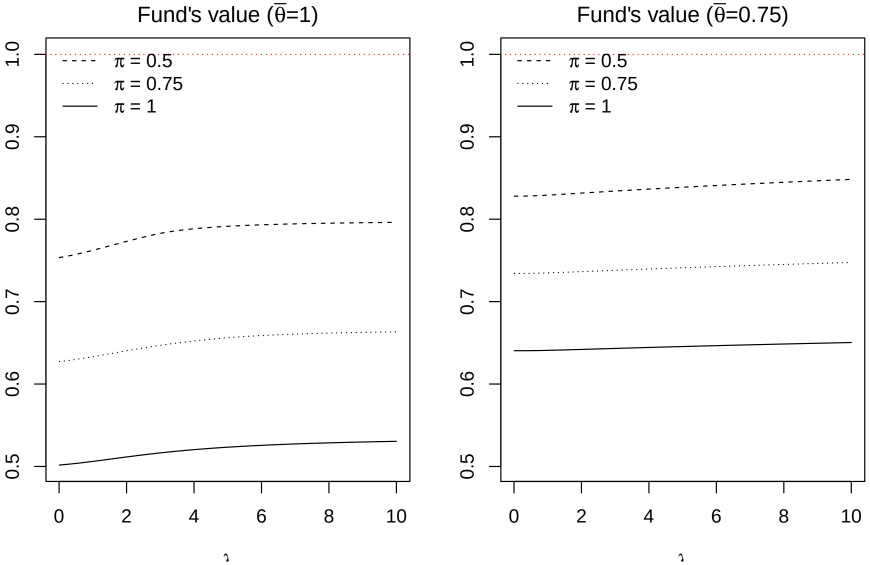
Important results (3/3): mismatch reduction

- All previous results still apply when $\overline{\theta} < 1$ but the PD is now decreasing with π.
- The expected portfolio value is decreasing in $\overline{\theta}$. [Hombert & Thesmar 2014]

Graphical illustration (1/2)



Graphical illustration (2/2)



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Description of the data

We use the Lipper TASS database from 2000 to 2014 where we select the intersection of:

- Funds whose NAV is in USD, with monthly frequency.
- Funds reporting their AUM monthly.
- Funds that are not FoF or multiple management classes.

$\Longrightarrow$ We concentrate in the current application on the management style *long/short equity*, though we have 8 others.

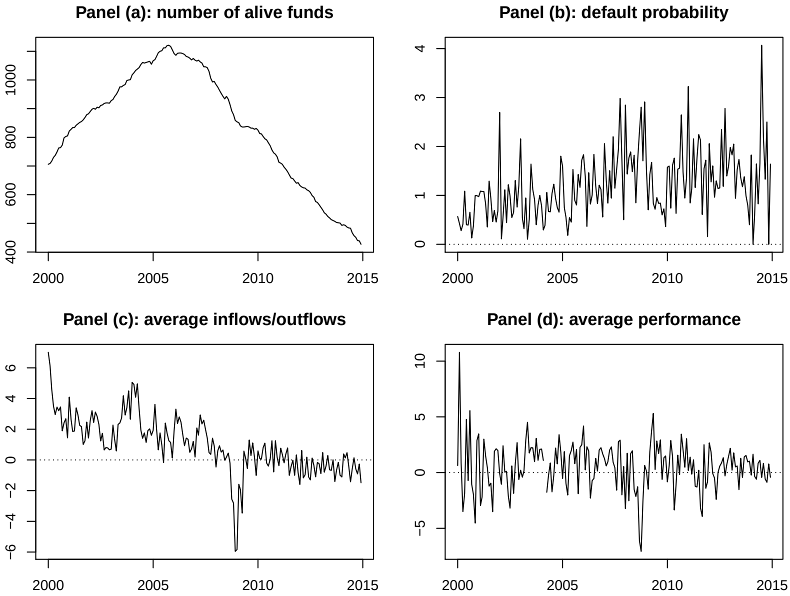
Data series

Default: $d_{j,t} = 1$ if j^{th} fund's NAV is not reported at t whereas it was reported before.

Flows: $f_{j,t}$ are the growth rate of AUM minus the performance.

Performance: Growth rate of the NAV.

The data (2000-2014)



Estimated quantities

We parameterize the following quantities:

- $\rho_t = \overline{\rho}$ is the average performance of the funds not experiencing outflows (cross-section avg first, time-series avg then).
- π_t is the cross-section average of number of funds experiencing outflows at period t.
- $\overline{\theta}_t$, λ_t and δ_t^* are estimated with a GMM-type estimator.

Cost functions

- $\overline{\theta}_t$ is bigger and as close as possible to the maximum observed outflow across funds.
- λ_t is positive and minimizes the distance between the model-implied and the measured default probability.

Formalization

- n_t is the number of alive funds at t, $f_{j,t}$ is the observed flows of fund j at t.
- The cost function of $\overline{\theta}_t$ is given by:

$$m_1 = \left(\overline{\theta}_t - \max_{j \in \{1,\dots,n_t\}} (-f_{j,t})\right)^2 \quad \text{with} \quad \overline{\theta}_t \geqslant \max_{j \in \{1,\dots,n_t\}} (-f_{j,t})\,.$$

- The cost function of λ_t is given by:

$$m_2 = \left(\frac{\pi_t \left[\overline{\theta}_t - \delta^*(\overline{\theta}_t, \lambda_t, \pi_t, \overline{\rho})\right]^{\lambda_t+1}}{\overline{\theta}_t(\lambda_t+1)\left[1 - \delta^*(\overline{\theta}_t, \lambda_t, \pi_t, \overline{\rho})\right]^{\lambda_t}} \exp\left(-\lambda_t \overline{\rho}\right) - \frac{1}{n_{t-1}} \sum_{j=1}^{n_{t-1}} d_{j,t} \right)^2 .$$

Algorithm: for each t, pick $(\overline{\theta}_t, \lambda_t)$, maximize the expected fund value to get δ^*, compute $\mathcal{C} = \omega m_1 + (1-\omega) m_2$ and minimize.

Products of estimation

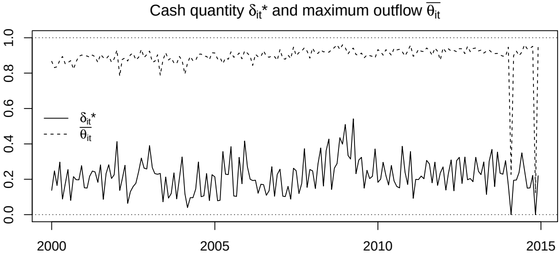
With the estimation, we get:

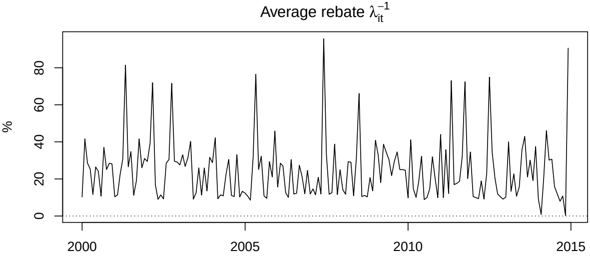
- A time-series for the maximum funding liquidity shock $\overline{\theta}_t$,
- A time-series for the optimal cash-holdings δ_t^*,
- A time-series for the average market liquidity λ_t,
- A risk factor taking both funding and market liquidity risks into account, which is computed as:

$$f_t = \overline{\rho} - \frac{1}{2} \log \mathbb{E}\left[F(\delta_t^*, \overline{\theta}_t, \lambda_t, \overline{\rho})\right]$$

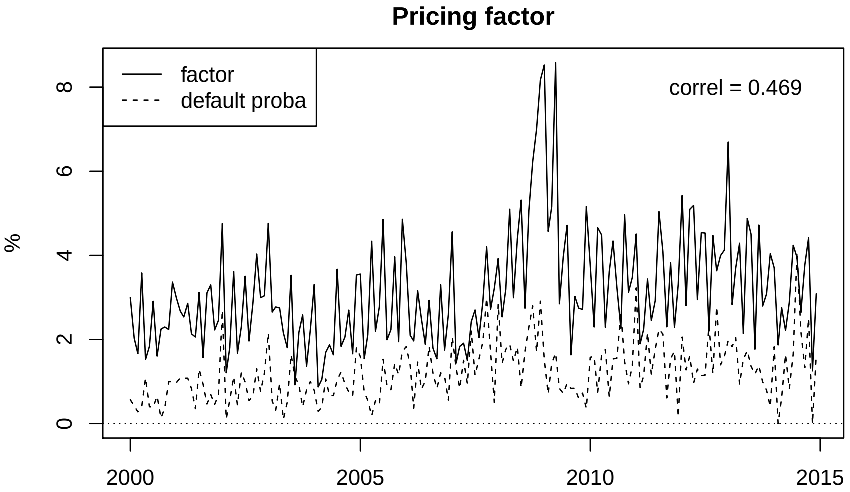
f_t is homogeneous to a one-period net return.

Long/short equity results (1/2)





Long/short equity results (2/2)



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Further empirical developments

- Use the factor in asset pricing regression to show that both market and funding liquidity shocks are priced in the hedge fund returns.
- Develop the same exercise on the different management style we possess.

What we have done:

- We developed a simple structural model to relate fund's optimal portfolio composition to **both** funding and market liquidity conditions.
- We produce a set of testable conditions for the fund's optimal portfolio.
- We take the model to the data and construct liquidity indicators per management style.