

Strategic Interaction between Hedge Funds and Prime Brokers

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- **Hedge fund (HF):** very reactive, large fluctuations, high leverage
- Leverage implies vulnerability
- Leverage is generated by prime brokers (borrowing) and off balance-sheet positions (derivatives)
- **Prime broker (PB):** very concentrated industry
 - ▶ Top 3: 41.1% of HF
 - ▶ 50% of HF have 1 PB, 3% of HF have more than 2 PB
- Prime brokers provide financing and services
- Interaction between HF and PB: mutually risky to each other
 - ▶ Large loss of HF → large loss of PB (asset liquidity risk)
 - ▶ PB can withdraw capital from HF (funding liquidity risk)
 - ▶ Main interaction: leverage decision

Model

Interaction between PB decision and HF financing strategy

- 1 representative PB and 1 representative HF
- Both are risk neutral and maximize the expected RoE for next period
- HF obtains financing for long and short positions through margin account (collateralized)

Model determines optimal decisions of HF and PB

- HF: amount of free cash and long/short balance
- PB: amount of free cash, margin rates, rehypothecation rate
- Equilibrium lending rate: risk-free rate + risk premium

Literature

Performance of hedge funds and their service providers:

- Olaru (2006), Klaus and Rzepkowski (2009), Cumming et al. (2013), Hespeler and Witt (2014), Mirabile (2015), Chung and Kang (2016)

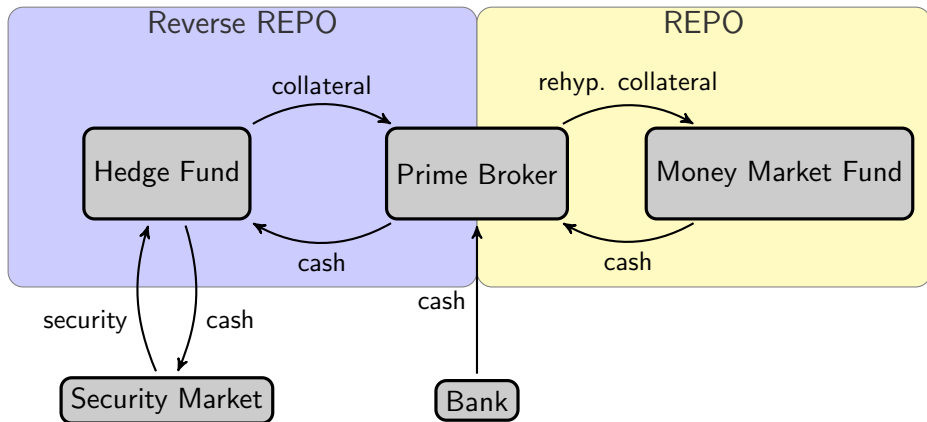
Leverage of hedge funds:

- McGuire and Tsatsaronis (2008), Duffie et al. (2009), Dai and Sundaresan (2010), Ang et al. (2011), Lan et al. (2013), Buraschi et al. (2014), Farnsworth (2014)

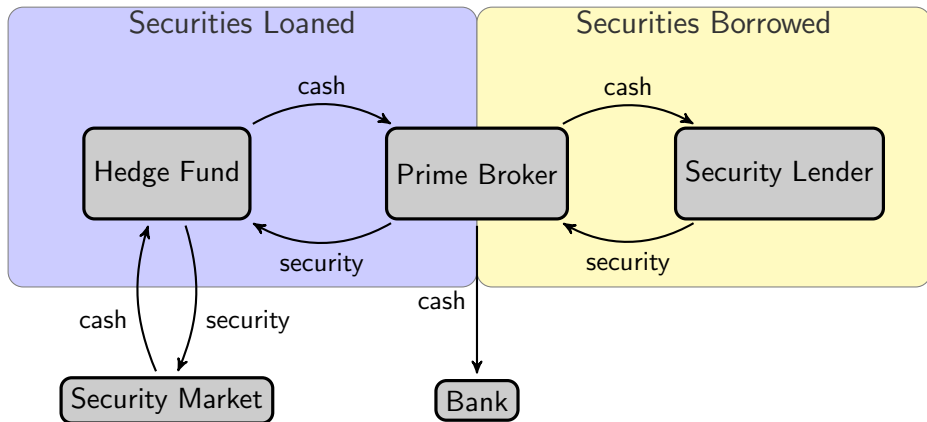
Leverage of intermediaries and funding liquidity risk:

- Adrian and Shin (2010), Dudley and Nimalendran (2011), Liu and Mello (2011), Adrian and Shin (2014), Adrian et al. (2014)

Financing HF Long Positions



Financing HF Short Positions



Framework

- HF optimization problem
- PB optimization problem
- Equilibrium lending rate

Mechanics of Leveraging: Long Positions

Main steps:

- HF buys n_t^+ shares at price p_t
- ... with $n_t^+ \mu_t^+$ of its own equity (margin: μ_t^+)
- ... and $n_t^+ l_t$ of PB loan
- Securities are deposited as collateral

Balance sheet:

- $L_t = \sum_j n_{j,t}^+ p_{j,t}$: long position financed through buying on margin
- $M_t^+ = \sum_j n_{j,t}^+ \mu_{j,t}^+$: margin account on long positions

Mechanics of Leveraging: Short Positions

Main steps:

- HF sells n_t^- shares at price p_t borrowed from the PB
- ... with $n_t^- \mu_t^-$ of its own equity (margin: μ_t^-)
- HF sells the securities on market
- ... and receives proceeds in cash, deposited as collateral

Balance sheet:

- $S_t = \sum_j n_{j,t}^- p_{j,t}$: short position
- $M_t^- = \sum_j n_{j,t}^+ \mu_{j,t}^-$: margin account on short positions

HF Balance Sheet

Assets		Liabilities and Equity	
Free cash	(C_t)	Debt	(D_t)
Cash proceeds	(P_t)	Short securities	(S_t)
Long securities	(L_t)	Equity	(N_t^H)

Margin account

Long positions

- Initial margin: $M_t^+ = \mu_{L,t}L_t = L_t - D_t$
- At $t + 1$: $M_{t+1}^+ = L_{t+1} - D_{t+1}$
- Maintenance margin: $M_{t+1}^+ \geq m_{L,t}L_{t+1}$

Short positions

- Initial margin: $M_t^- = \mu_{S,t}S_t = P_t - S_t$
- At $t + 1$: $M_{t+1}^- = P_{t+1} - S_{t+1}$
- Maintenance margin: $M_{t+1}^- \geq m_{S,t}S_{t+1}$

Portfolio margining

$$M_{t+1} = M_{t+1}^+ + M_{t+1}^- = \max(L_{t+1} - D_{t+1} + P_{t+1} - S_{t+1}, m_{L,t}L_{t+1} + m_{S,t}S_{t+1})$$

HF Optimization Program

- risk neutral \Rightarrow max RoE, $r_{NH,t+1} = N_{t+1}^H / N_t^H - 1$
- control variables
 - ▶ cash holdings: $(1 - \alpha_t)N_t^H$
 - ▶ risky strategy:
 - ★ long position: $\alpha_t \gamma_t N_t^H$
 - ★ short position: $\alpha_t (1 - \gamma_t) N_t^H$
- simple investment strategy: exposition to market risk
 - ▶ $r_{L,t+1} = \beta_L r_{M,t+1}$
 - ▶ $r_{S,t+1} = \beta_S r_{M,t+1}$

Parametrized HF Balance Sheet

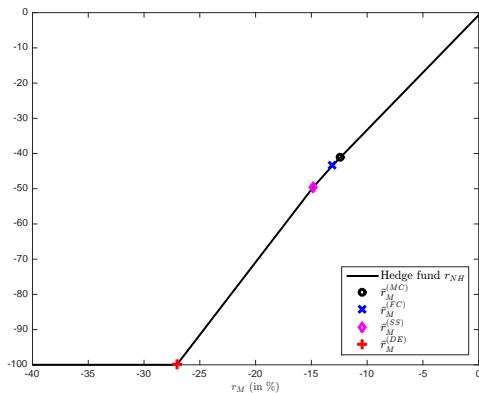
Assets			Liabilities and Equity		
(r_F)	Free cash	$(1 - \alpha_t)$	Margin debt	$\alpha_t \gamma_t \frac{1 - \mu_{L,t}}{\mu_{L,t}}$	(r_D)
(r_F)	Cash proceeds	$\alpha_t (1 - \gamma_t) \frac{1 + \mu_{S,t}}{\mu_{S,t}}$	Short securities	$\alpha_t (1 - \gamma_t) \frac{1}{\mu_{S,t}}$	(r_S)
(r_L)	Long securities	$\alpha_t \gamma_t \frac{1}{\mu_{L,t}}$	Equity	1	(r_{NH})
	- Free cash	$(1 - \alpha_t)$	- Long account	$\alpha_t \gamma_t$	
	- Long account	$\alpha_t \gamma_t$	- Short account	$\alpha_t (1 - \gamma_t)$	

(scaled by N_t^H)

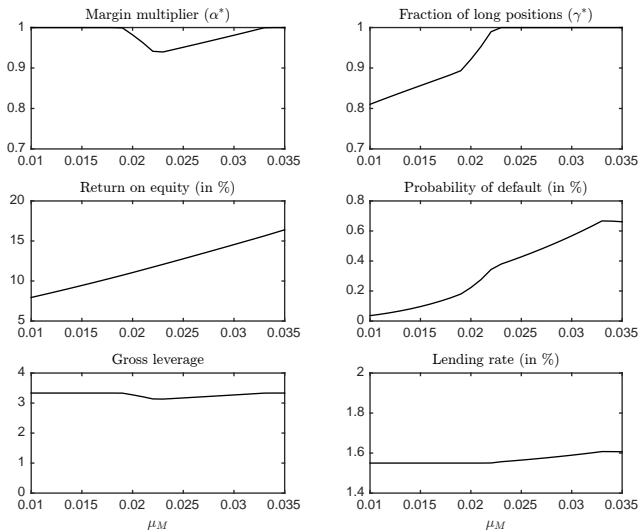
Impact of a Market Crash

When market goes down, sequence of HF actions (increasing cost):

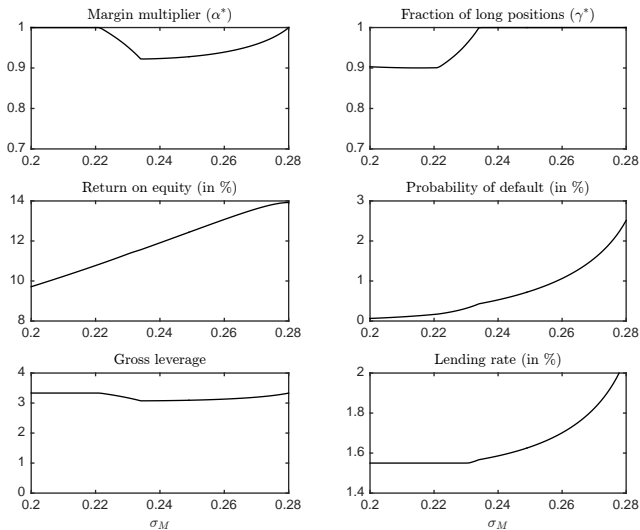
- ↓
- Regular margin call
 - Using free cash only
 - Buying back short positions
 - Selling long positions
 - Default



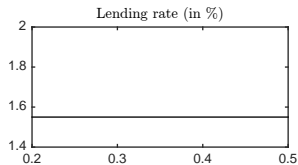
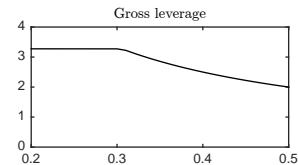
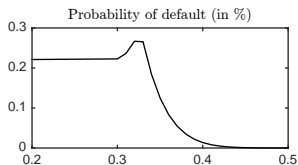
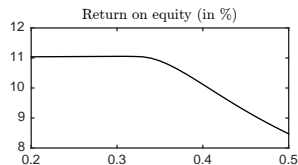
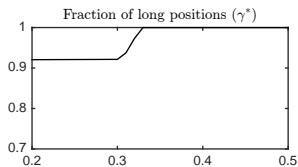
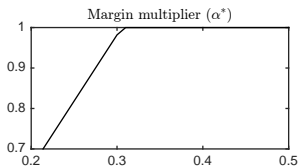
HF Decisions as a Function of μ_M



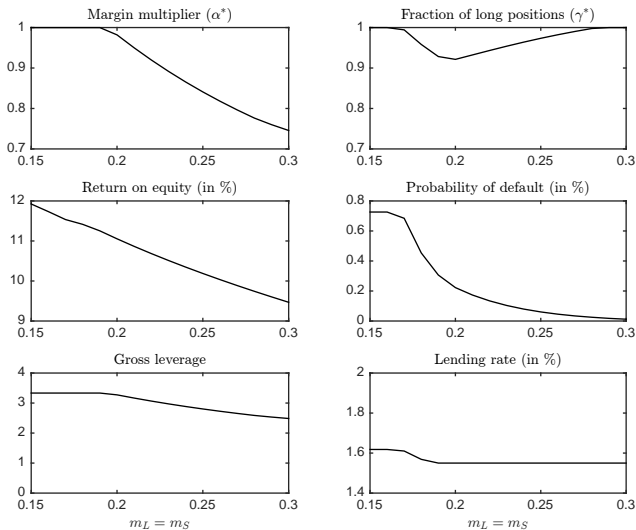
HF Decisions as a Function of σ_M



HF Decisions as a Function of $\mu_L = \mu_S$

 $\mu_L = \mu_S$ $\mu_L = \mu_S$

HF Decisions as a Function of $m_L = m_S$



Sources of PB Revenues

PB makes money from HF through 3 channels:

- the difference between the borrowing rate and the lending rate for HF long positions
- the difference between cost and revenue of financing HF short positions
- the remuneration on the interbank market

PB Actions

Buying on margin

- PB provides $[\alpha_t \gamma_t (1 - \mu_{L,t}) / \mu_{L,t}] N_t^H$ of cash to the HF
- Its own financing:
 - ▶ $[\rho_t \alpha_t \gamma_t (1 - \mu_{L,t}^P) / \mu_{L,t}] N_t^H$ from rehypothecation
 - ▶ $[\alpha_t \gamma_t ((1 - \mu_{L,t}) - \rho_t (1 - \mu_{L,t}^P)) / \mu_{L,t}] N_t^H$ from the (unsecured) interbank market

Short selling

- PB borrows securities from another broker and deposits $[\alpha_t (1 - \gamma_t) (1 - \mu_{S,t}^P) / \mu_{S,t}] N_t^H$ as collateral
- It lends the securities to the HF and receives $[\alpha_t (1 - \gamma_t) (1 - \mu_{S,t}) / \mu_{S,t}] N_t^H$ as collateral (cash proceeds)
- The difference, $[\alpha_t (1 - \gamma_t) (\mu_{S,t} - \mu_{S,t}^P) / \mu_{S,t}] N_t^H$, is invested on the (unsecured) interbank market.

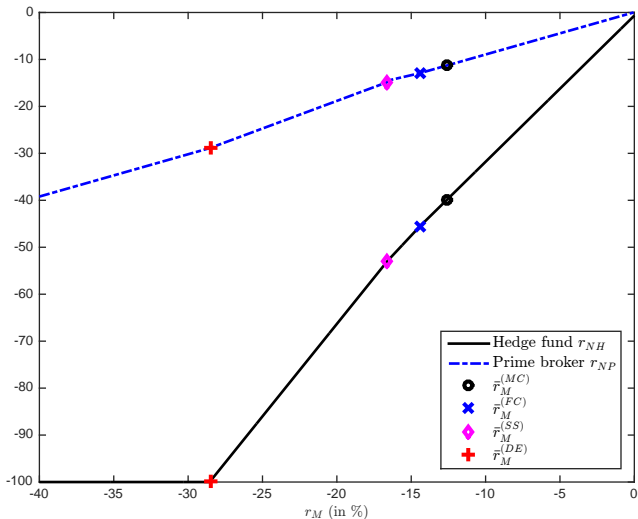
PB Optimization Program

- risk neutral \Rightarrow max RoE, $r_{NP,t+1} = N_{t+1}^P / N_t^P - 1$
- control variables
 - ▶ cash holdings: $(1 - \alpha_t^P)N_t^P$
 - ▶ rehypothecation rate: ρ_t
 - ▶ margin rates: $\mu_{L,t}, \mu_{S,t}, m_{L,t}, m_{S,t}$
 - ▶ own securities: A_t^P
- N_t^P determined by regulatory limit on leverage: Debt $\leq \vartheta$ Equity
- If HF liquidates its positions, PB also has to liquidate (with a cost)

PB Balance Sheet

Assets	Liabilities and Equity
Unsecured cash	Unsecured borrowing
$(r_I) \quad \alpha_t(1 - \gamma_t) \frac{\mu_{S,t} - \mu_{S,t}^P}{\mu_{S,t}} N_t^H + (1 - \alpha_t^P) N_t^P$	$\alpha_t \gamma_t \frac{1 - \mu_{L,t} - \rho_t(1 - \mu_{L,t}^P)}{\mu_{L,t}} N_t^H \quad (r_I)$
Collateralized Agreements: (Securities borrowed)	Collateralized Financings: (Securities loaned)
$(r_F) \quad \alpha_t(1 - \gamma_t) \frac{1 + \mu_{S,t}^P}{\mu_{S,t}} N_t^H$	$\alpha_t(1 - \gamma_t) \frac{1 + \mu_{S,t}}{\mu_{S,t}} N_t^H \quad (r_F)$
Receivables from HF: (Reverse repo)	Payables to MMF: (Repo)
$(r_D) \quad \alpha_t \gamma_t \frac{1 - \mu_{L,t}}{\mu_{L,t}} N_t^H$	$\rho_t \alpha_t \gamma_t \frac{1 - \mu_{L,t}^P}{\mu_{L,t}} N_t^H \quad (r_C)$
Own securities portfolio	Equity
$(r_M) \quad A_t^P = \alpha_t^P N_t^P$	$N_t^P \quad (r_{NP})$
	- Free cash $(1 - \alpha_t^P) N_t^P$
	- Other $\alpha_t^P N_t^P$

Impact of a Market Crash



Equilibrium Lending Rate

- Lending rate = risk-free rate + risk premium

$$r_{D,t} = r_{F,t} + RP_t$$

- RP_t covers the expected loss of the PB due to a HF default:

$$\begin{aligned} RP_t &= E_t \left[\frac{(1 + r_{D,t})D_t^H - (D_{t+1}^H | \text{HF default})}{D_t^H} \right] \\ &= \left[(1 + r_{D,t}) - (1 + r_{F,t}) \frac{[\alpha_t(1 - \gamma_t) + \mu_{S,t}(1 - \alpha_t\gamma_t)]\mu_{L,t}}{\alpha_t\gamma_t(1 - \mu_{L,t})\mu_{S,t}} \right. \\ &\quad \left. - (1 + \beta_L\mu_{M,t}^{(DE)}) \frac{1 - \phi}{1 - \mu_{L,t}} + (1 + \beta_S\mu_{M,t}^{(DE)}) \frac{(1 + \theta)\alpha_t(1 - \gamma_t)\mu_{L,t}}{\alpha_t\gamma_t(1 - \mu_{L,t})\mu_{S,t}} \right] F(\log(1 + \bar{r}_{M,t}^{(DE)})) \end{aligned}$$

- RP_t depends on
 - ▶ HF decisions (α and γ)
 - ▶ PB decisions (μ_L and μ_S)

Calibration

	Symbol	Value
Hedge fund		
Liquidation cost for long positions	ϕ	5%
Liquidation cost for short positions	θ	2%
Sensitivity of long positions	β_L	1.1
Sensitivity of short positions	β_S	0.2
Prime broker		
Liquidation cost for long positions	ϕ'	2.5%
Liquidation cost for short positions	θ'	1%
Initial margin rate	$\mu_L^P = \mu_S^P$	20%
Maintenance margin rate	$m_L^P = m_S^P$	10%
Maximum leverage ratio	ϑ	10
Market return and interest rates		
Expected market return	μ_M	2%
Market volatility	σ_M	22.5%
Risk-free rate	r_F	1.5%
General collateral rate	r_C	1.55%
Short-term interbank rate	r_I	1.75%

Benchmark

	$\vartheta = 10$
	$\sigma_M = 22.5\%$
HF decision variables	
α^*	0.923
γ^*	0.902
PB decision variables	
$\mu_L^* = \mu_S^*$	0.264
$m_L^* = m_S^*$	0.18
α^{P^*}	0.736
ρ^*	0.698
Rates and expected returns (% per year)	
$r_{D,t}^*$	1.552
$E_t[r_{NH,t+1}]$	11.458
$E_t[r_{NP,t+1}]$	3.351
Probability of default (in %)	
$\Pr[r_{M,t+1} < \bar{r}_{M,t+1}^{(DE)}]$	0.373

Balance sheet of HF at equilibrium

Assets		Liabilities and Equity	
Free cash	1153	Margin debt	34898
Cash proceeds	6498	Short securities	5142
Long securities	47390	Equity	15000
		- Free cash	1153
		- Long account	12492
		- Short account	1356
Total	55040	Total	55040

Balance sheet of PB at equilibrium

Assets		Liabilities and Equity	
Unsecured Cash	1246	Unsecured borrowing	1806
Securities borrowed	6171	Securities loaned	6498
Receivables from HF	34898	Payables to investor	26473
Securities portfolio	2559	Equity	3478
		- Free cash	3478
		- Other	919
Total	44873	Total	44873

Alternative parametrizations

	Benchmark $\vartheta = 10, \sigma_M = 22.5\%$	Alternative 1 $\vartheta = 5$	Alternative 2 $\sigma_M = 23\%$
Hedge fund's decision variables			
α^*	0.923	0.945	0.936
γ^*	0.902	0.924	0.958
Prime broker's decision variables			
$\mu_L^* = \mu_S^*$	0.264	0.254	0.292
$m_L^* = m_S^*$	0.180	0.154	0.197
α^{P^*}	0.736	0.677	0.695
ρ^*	0.698	0.556	0.709
Rates and expected returns (% per year)			
$r_{D,t}^*$	1.550	1.648	1.550
$E_t[r_{NH,t+1}]$	11.458	11.832	11.414
$E_t[r_{NP,t+1}]$	3.351	3.590	3.167
Probability of default (in %)			
$\Pr[r_{M,t+1} < \bar{r}_{M,t+1}^{(DE)}]$	0.373	0.893	0.348

Strategic Interaction

- **More regulation on PB (decrease ϑ)**

- ▶ PB reduces margin rate
- ▶ HF increases leverage
- ▶ Prob of default increases

⇒ More risk

- **Riskier market return (increase σ)**

- ▶ PB increases margin rates
- ▶ HF decreases leverage
- ▶ Prob of default decreases

⇒ Less risk

Extensions

- impact of Basel III ratios
- redemption risk from HF investors
- investment strategy of the HF
-