Volatility Risk and Expected Returns

Philippe Mueller
WBS

Paul Whelan
CBS

Ecole Hoteliere Lausanne
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Motivation

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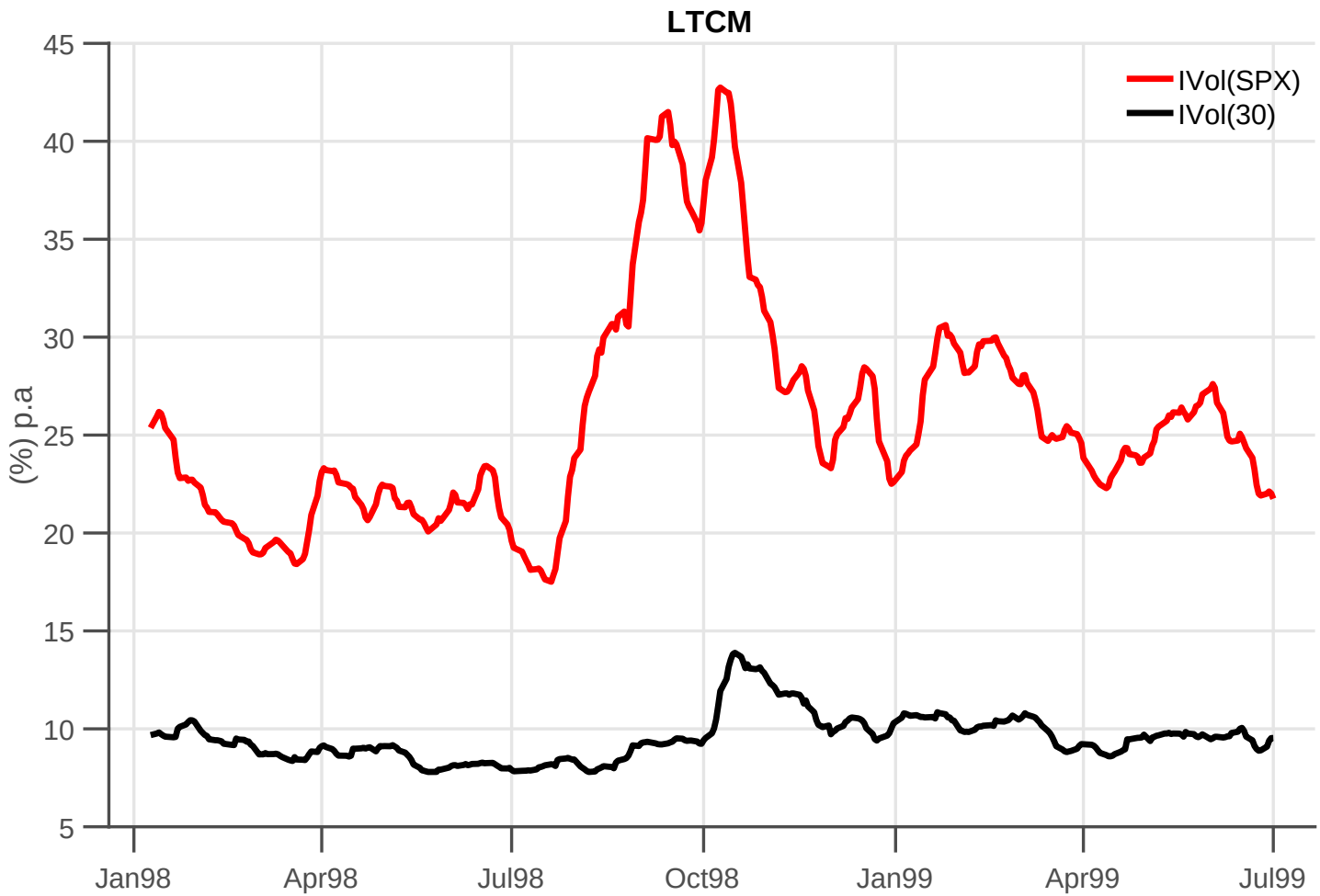

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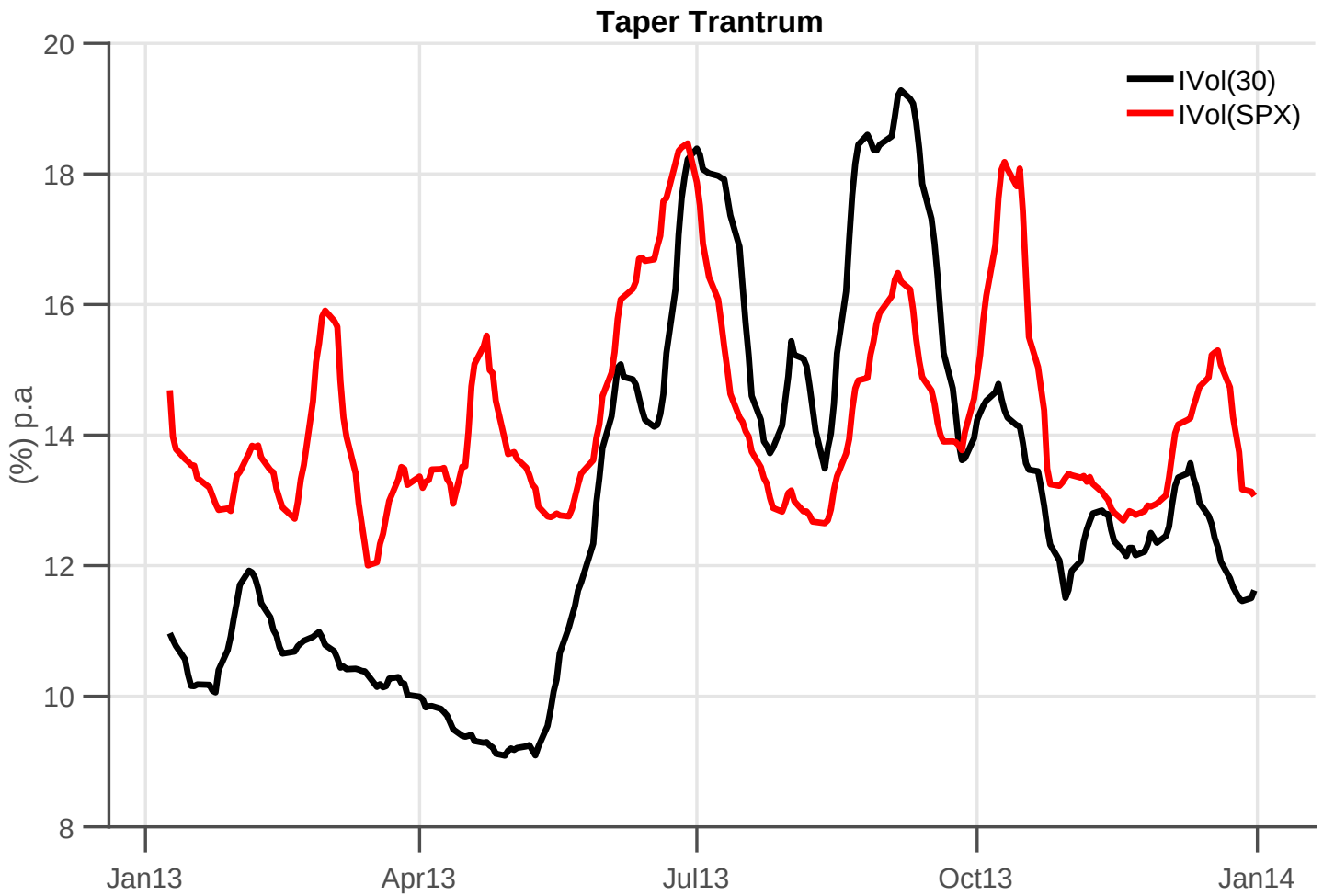

Motivation

- **220% rise** in equity market IVol around the LTCM crisis.
- **180% rise** in implied bond market IVol around the Taper Tantrum.
- **Heightened variance (volatility)** is associated with adverse economic conditions and **uncertainty** in financial markets. **Compensation** for these risks is given by **variance risk premia (VRP)**.
- Large market for variance products in the equity space. Hard to find such products in **fixed income**.
- Research agenda:
  - Construct **ex ante** variance risk premia for Treasuries and equities using high frequency data,
  - document stylized facts about **dynamics** of variance risk premia
  - examine the **predictive power** of variance risk premia for stock and bond excess returns.
  - Ultimately, would like to understand bond and equity variance risk premia within an **equilibrium** context.

Research questions

Q1 What is the relationship between expected returns and implied variance?

Q2 Do variance risk premia contain additional information about expected returns w.r.t implied variance?

Q3 Is there a link between the joint dynamics of expected returns across stock and bonds, and compensation for variance risk?

Q4 Could consumption based models be consistent with the joint dynamics of compensation for variance risk and expected returns?

Findings

Q1 We conduct a series of tests that show a positive but weak link between variance and expected returns.

Q2 replacing implied variance by variance risk premia in predictability regressions dramatically improves results:

- $VRP(equity)$ generates return predictability on equity at short horizons but does not predict bonds at any horizon.
- $VRP(STB)$ generates return predictability on all bonds at long horizons.
- $VRP(LTB)$ generates return predictability on equity at both short and long horizons.

Q3 Combing these observations we show a single spread factor,

$$Spread(TVRP) = VRP(LTB) - VRP(STB),$$

summarises the predictable variation in stock and bond markets across all horizons.

Q4 (a) factors that capture consumption risks explain well $Spread(TVRP)$ but (b) we also find it is related to the capital ratio of primary dealers.

Variance risk premia

Measuring variance risk premia

We use the following definition of the variance risk premium:

$$\text{VRP}_t = \mathbb{E}_t^{\mathbb{P}}\left(\int_t^{t+1} \sigma_s^2 ds\right) - \mathbb{E}_t^{\mathbb{Q}}\left(\int_t^{t+1} \sigma_s^2 ds\right)$$

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Measuring variance risk premia

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- Prima facie, it is not clear how to measure the ex ante variance risk premium.
  - While the integrated variance under the **risk-neutral** measure can be captured via option prices ...
  - there are potentially many ways to capture the expected realized variance under the **physical** measure.
  - Moreover, the integrated variance is a theoretical construct and not observable in practice.

Estimation of expected realized variance

We use a projection method via the **jump robust HAR-TCJ-RV model** [Corsi, Pirino, and Renò, 2010]:

- First, consider the following augmented HAR-TCJ-RV model:

$$\begin{aligned}\ln \widetilde{RV}_{t+21,M} = \quad & \alpha + \beta_{C,D} \ln \widehat{TC}_{t,D} + \beta_{J,D} \ln(1 + \widehat{TJ}_{t,D}) + \beta_{C,W} \ln \widehat{TC}_{t,W} \\ & + \beta_{C,M} \ln \widehat{TC}_{t,M} + \beta_{IV,0} \ln IV_t + \beta_{IV,1} \ln IV_{t-1},\end{aligned}$$

  where $\widehat{TJ}_t = I_{\{C-Tz>\Phi_{0.99}\}} (RV_t - TBPV_t)^+$ and $\widehat{TC}_t = RV_t - \widehat{TJ}_t$. h is the forecast horizon in days.

- We then implement the regression using an expanding window. This allows us to obtain **real-time forecasts** at time t for a one month horizon, $\widetilde{RV}_{t+21,M}$ using the time-varying parameter estimates.

- $\widetilde{RV}_{t,M}$ is the one month realized variance using the definition in Bondarenko (2014), instead of the standard definition that uses the sum of squared intradaily returns.

Generalized variance swap: Fixed leg

- Measure the expected variance under the risk-neutral measure using the **fair strike** of a Generalized variance swap [Choi, Mueller & Vedolin (2016)] for Treasuries and [Bondarenko (2014)] for equities.
- The contract is robust to jumps and can be perfectly hedged using a discrete partition as long as $\widetilde{RV}_{t,h}$ is used to calculate the contract payoff.
- The fair strike is given by a portfolio of put and call options:

$$\mathbb{E}_t^{\mathbb{Q}_T}\left[\int_0^T \sigma_t^2\right] \equiv \tilde{F}_{t,T} = \frac{2}{p_{t,T}}\int_0^{F_{t,T}} \frac{P_{t,T}(K)}{K^2}dK + \int_{F_{t,T}}^{\infty} \frac{C_{t,T}(K)}{K^2}dK$$

Stylized facts

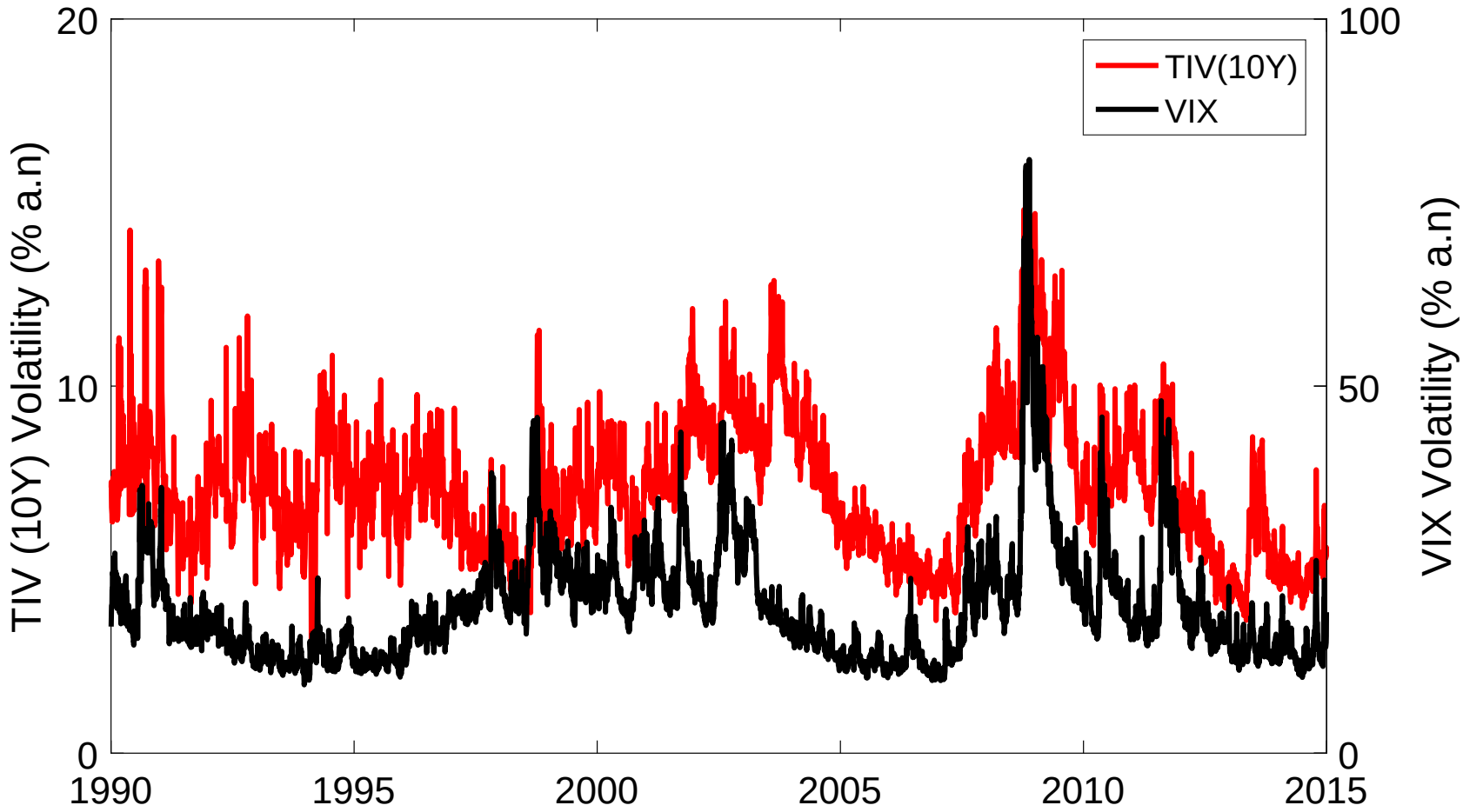
Data

- **High frequency futures** data from the Chicago Board of Trade (CBOT/CME):
  - Futures on **10 year** Treasury securities and **S&P 500 index**.
  - Intra-day price data (electronic and pit trades) during regular trading hours (7:20AM to 2:00PM).
  - Very liquid market with over 4m contracts traded per day for interest rate futures and roughly 2.5m contracts for index futures.
- **Daily options** data from the Chicago Board of Trade (CBOT/CME):
  - End of day option data matched to corresponding futures.
  - Also very liquid with around 1.5m contracts per day for Treasury options and 350k contracts for index options.

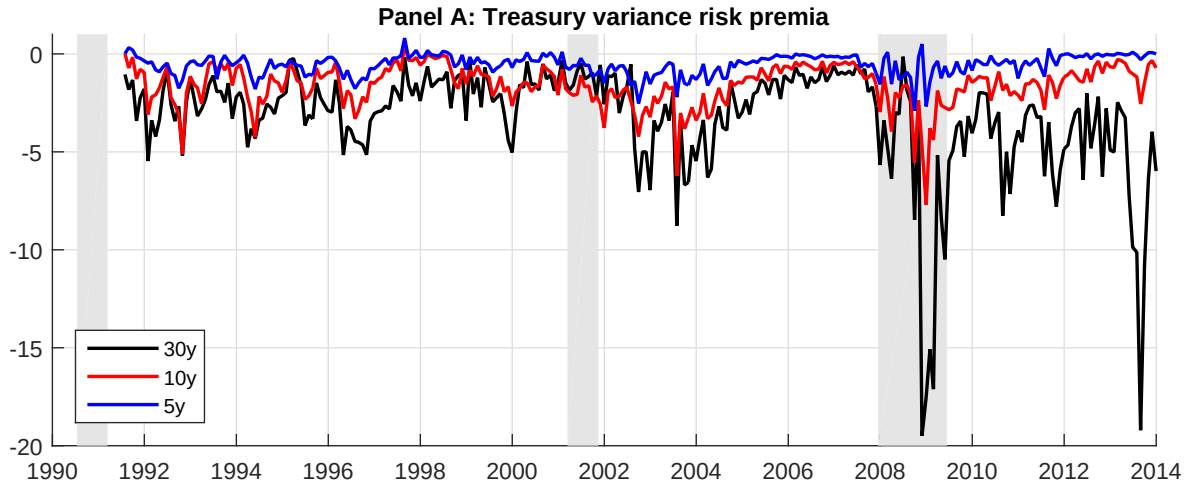
Summary statistics for volatilities

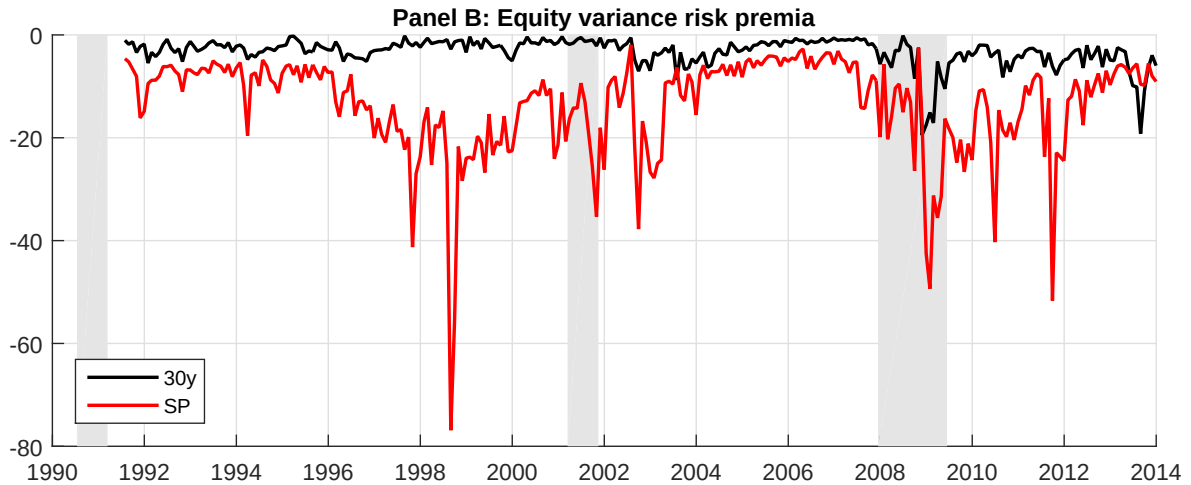
	S&P 500 index	5yr Bond	10yr Bond	30yr Bond
		Physical Volatility		
Mean	15.89	4.02	5.96	9.30
Std Dev	7.36	0.93	1.26	2.05
Min	6.96	1.59	3.17	5.70
Max	59.64	7.68	10.62	18.69
Skew	2.07	0.29	0.94	1.57
Kurt	10.07	3.97	5.01	6.74
AR(1)	0.85	0.84	0.84	0.86
		Risk Neutral Volatility		
Mean	20.18	4.69	7.32	11.07
Std Dev	7.90	1.28	1.71	2.70
Min	10.42	1.79	3.97	6.60
Max	59.89	9.69	14.33	24.15
Skew	1.63	0.23	0.77	1.70
Kurt	6.94	3.45	4.45	7.44
AR(1)	0.86	0.83	0.84	0.85

Treasury implied volatility and VIX



Treasury and equity VRP



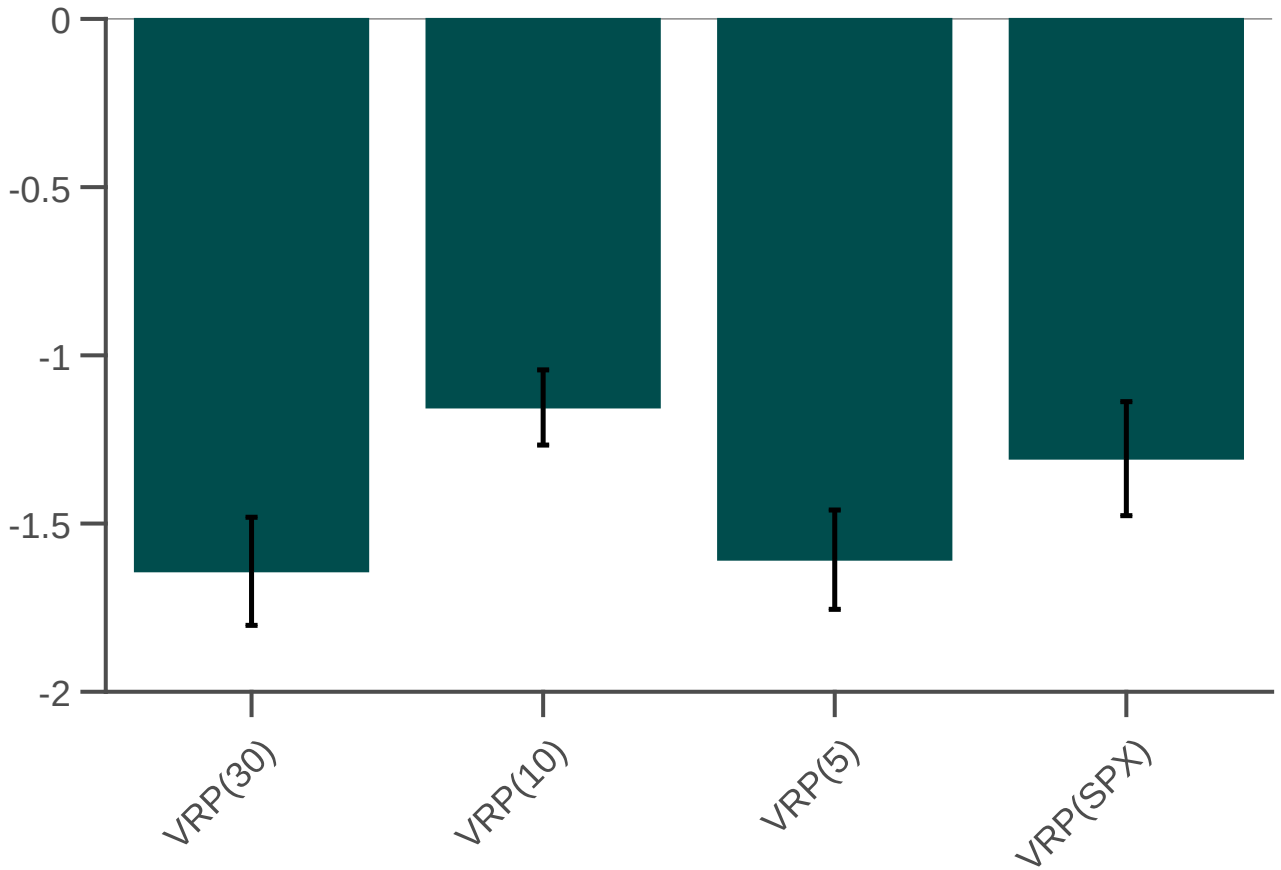


Summary statistics for VRP

	S&P 500 index	5yr Bond	10yr Bond	30yr Bond
	Variance Risk Premiums			
Mean	−13.32	−0.56	−1.57	−3.19
Std Dev	8.11	0.48	0.97	2.44
Min	−47.47	−2.49	−5.91	−15.94
Max	−0.68	0.41	0.06	−0.08
Skew	−1.45	−1.06	−0.99	−2.26
Kurt	5.47	4.17	4.30	10.01
AR(1)	0.86	0.80	0.82	0.86

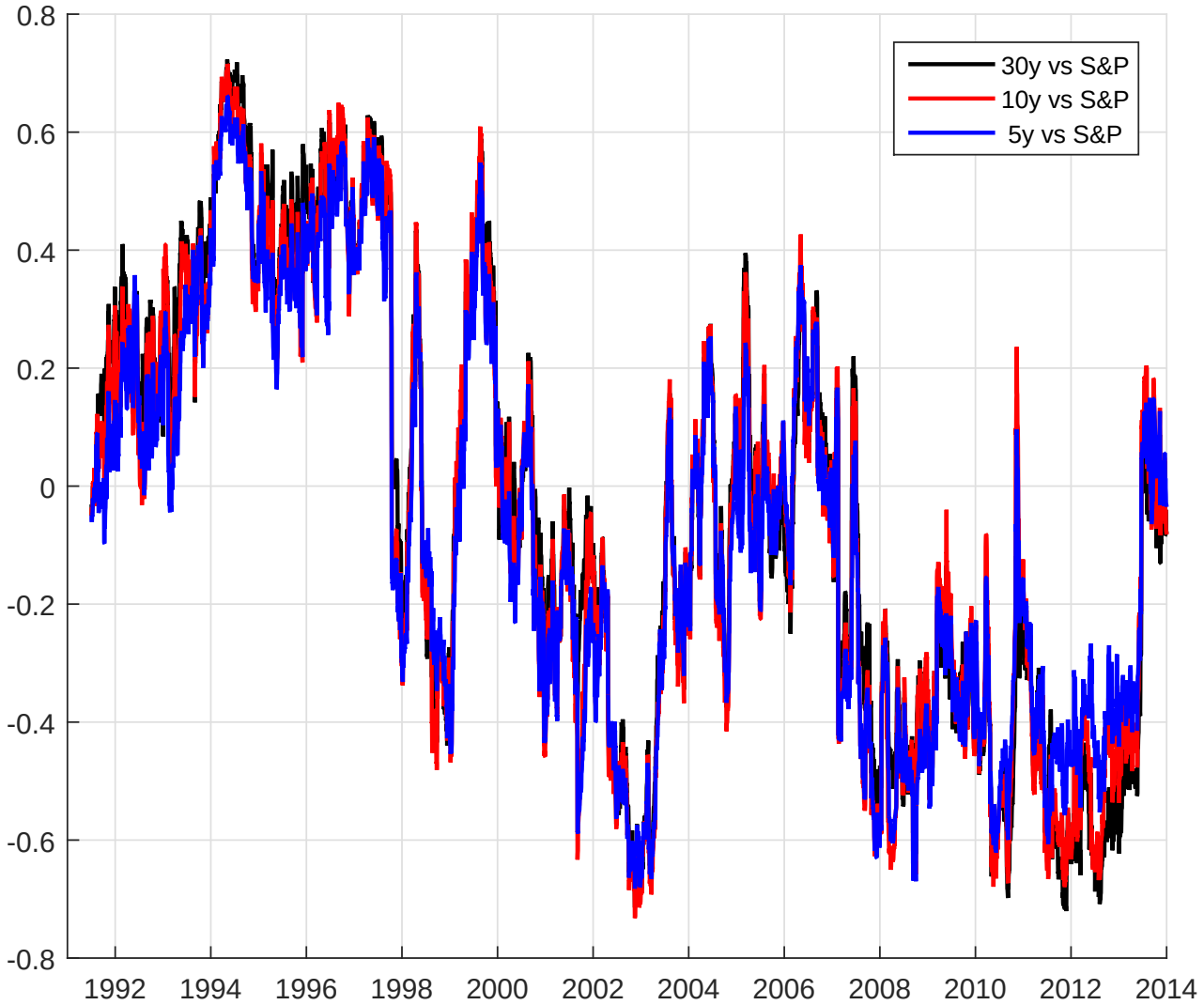
Variance Risk Premia estimates are in monthly squared percentage points.

Economic magnitudes

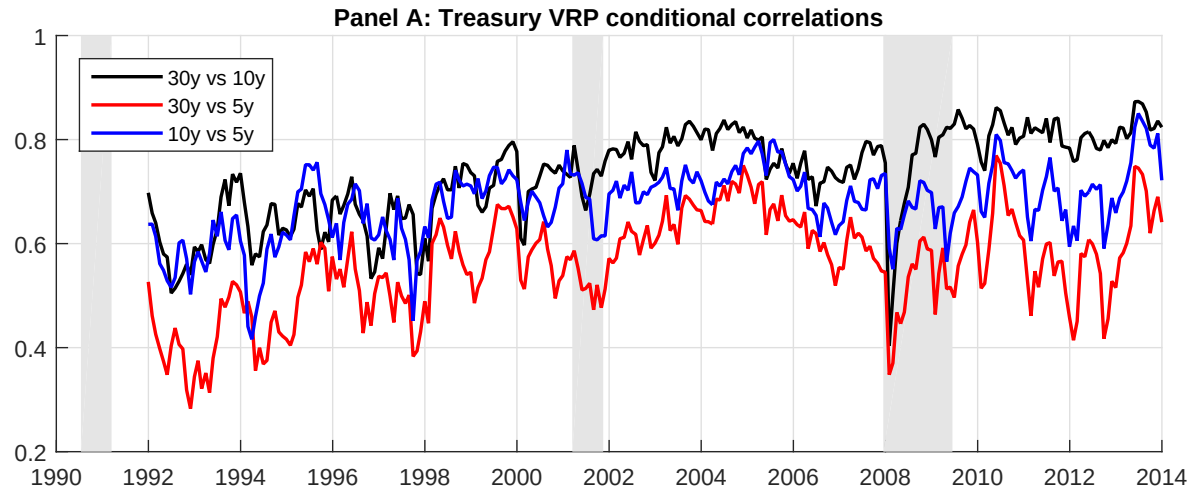


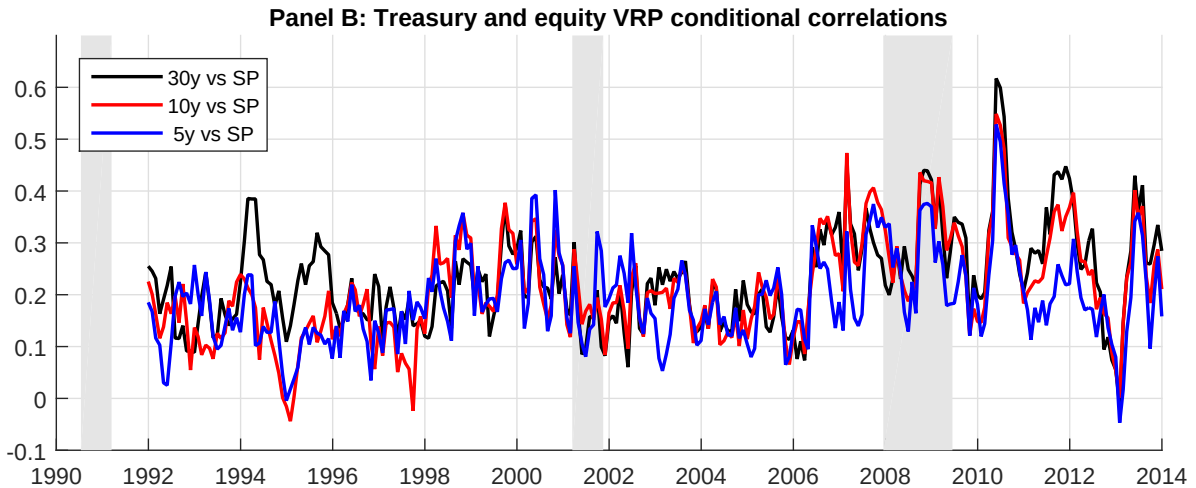
Relative ex-ante risks of trading variance measured by 'Sharpe Ratios'.

Stock-bond correlations



VRP correlations





Treasury variance risk premia stylized facts

- Implied and realized variances are **increasing** in the tenor. The 30y Treasury IV is about half the size of the VIX.
- Implied variance is higher than expected realized variance. This implies a **negative** Treasury variance risk premium on average for all tenors.
- Term structure of Treasury IV is **downward sloping** for both tenors on average, term structure of expected RV is roughly flat. This implies a negative but upward sloping term structure of variance risk premia.
- Ex ante results are consistent with the results for **realized** TVRP measured ex post.
- TVRP are **not spanned** by the term structure of interest rates but the **slope** of the term structure explains a significant fraction of the overall variation in TVRP for all tenors.
- VRP **correlations are time-varying**. Correlations among TVRP are increasing over time and are **high** on average. Correlations between EVRP and TVRP are slightly positive on average but occasionally turn **negative**.

Regression analysis

TIV and economic activity

- TIV **predicts economic activity** for a horizon up to eight months:

$$\text{CFNAI}_{t+n} = \beta_n^{\text{TIV}} \text{TIV}_t^2 + \beta_n^{\text{slope TIV}} \text{slope}_t^{\text{TIV}} + \ldots + \epsilon_{t+n}, \quad n = 0, \ldots, 12.$$

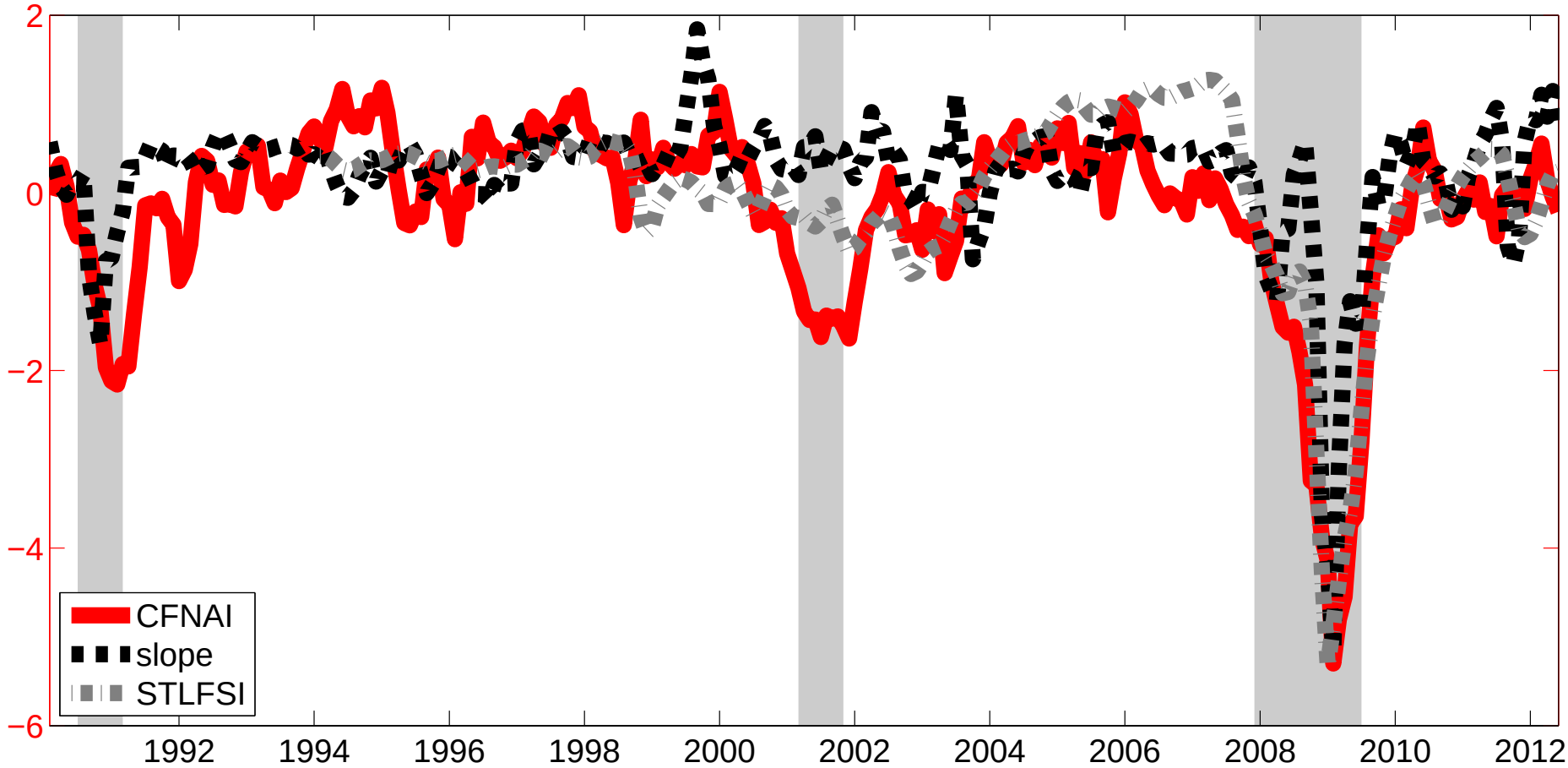
  - The contemporaneous correlation between the slope of TIV and CFNAI is 68%.
  - A 1 SD shock to the slope of TIV results in a 0.35 SD move in CFNAI for horizons up to five months (even more in univariate regressions).
  - Adding the VIX or the slope of the VIX does not drive out TIV.

- TIV **captures and predicts periods of stress** in financial markets:

$$\text{STLFSI}_{t+n} = \beta_n^{\text{TIV}} \text{TIV}_t^2 + \beta_n^{\text{slope TIV}} \text{slope}_t^{\text{TIV}} + \epsilon_{t+n}, \quad n = 0, \ldots, 12.$$

  - The contemporaneous correlation between the slope of TIV and the stress index is 76%.
  - A 1 SD shock to the slope of TIV results in a -0.5 SD move in the stress index.

TIV, CFNAI and STLFSI



Predictability CFNAI

horizon	0	2	4	6	8	10	12
TIV^2	-0.302 (-6.45)	-0.274 (-5.14)	-0.213 (-4.73)	-0.121 (-3.30)	-0.056 (-3.01)	-0.039 (-1.82)	-0.038 (-1.34)
Adj. R^2	28.89%	23.90%	14.48%	4.64%	0.98%	0.49%	0.47%
slope TIV	0.622 (3.11)	0.617 (3.35)	0.520 (3.57)	0.327 (3.61)	0.191 (2.16)	0.170 (1.54)	0.121 (0.99)
Adj. R^2	19.71%	19.33%	13.60%	5.32%	1.83%	1.41%	0.74%
TIV^2	-0.237 (-6.05)	-0.201 (-4.94)	-0.146 (-4.26)	-0.074 (-1.80)	-0.023 (-0.65)	-0.007 (-0.17)	-0.018 (-0.38)
slope TIV	0.330 (3.74)	0.366 (3.60)	0.335 (3.53)	0.232 (2.23)	0.162 (1.29)	0.160 (1.07)	0.098 (0.59)
Adj. R^2	32.84%	28.74%	18.40%	6.28%	1.57%	1.04%	0.43%
TIV^2	-0.172 (-3.43)	-0.069 (-1.43)	-0.079 (-1.77)	0.019 (0.31)	0.058 (0.93)	0.050 (0.74)	0.010 (0.13)
slope TIV	0.339 (3.01)	0.335 (2.77)	0.355 (3.05)	0.261 (2.01)	0.230 (1.49)	0.261 (1.37)	0.179 (0.84)
VIX^2	-0.064 (-1.22)	-0.124 (-2.40)	-0.069 (-1.28)	-0.097 (-1.58)	-0.096 (-1.41)	-0.080 (-1.27)	-0.046 (-0.72)
slope VIX	-0.037 (-0.32)	0.023 (0.22)	-0.066 (-0.53)	-0.101 (-0.71)	-0.200 (-1.08)	-0.263 (-1.34)	-0.210 (-1.09)
Adj. R^2	33.52%	35.37%	18.90%	7.66%	2.61%	2.69%	1.37%
TIV^2	-0.188 (-3.82)	-0.069 (-1.41)	-0.070 (-1.54)	0.039 (0.69)	0.089 (1.44)	0.090 (1.38)	0.054 (0.71)
slope TIV	0.309 (2.90)	0.335 (2.76)	0.372 (3.14)	0.298 (2.24)	0.286 (1.81)	0.333 (1.74)	0.261 (1.19)
VIX^2	-0.053 (-1.03)	-0.124 (-2.38)	-0.076 (-1.39)	-0.112 (-1.81)	-0.120 (-1.72)	-0.111 (-1.66)	-0.081 (-1.14)
slope VIX	-0.014 (-0.13)	0.024 (0.22)	-0.081 (-0.64)	-0.135 (-0.94)	-0.255 (-1.36)	-0.334 (-1.63)	-0.288 (-1.39)
slope yields	-0.099 (-1.73)	-0.001 (-0.01)	0.057 (0.87)	0.124 (1.65)	0.190 (2.17)	0.240 (2.41)	0.268 (2.38)
Adj. R^2	34.34%	35.13%	18.94%	8.96%	6.15%	8.64%	9.14%

IV/RV/VRP and excess returns

Q1 What is the relationship between expected returns and implied (realised) variance?

Q2 Do variance risk premia contain additional information about expected returns w.r.t implied (realised) variance?

- Roll over a fully collateralized futures positions
- S&P 500 / 2yr Bond / 5yr Bond / 10yr Bond / 30yr Bond
- Estimate return predictability regressions of the form

$$xrt_{t+h} = \alpha + \beta F_t + \epsilon_{t+h}$$
$$F_t = IV_t \text{ , } RV_t \text{ or } VRP_t$$

and $h =$ the horizon of the forecast

Univariate IV-predictability: 3-month forecasting horizon

	$IV(SPX)$	$IV(5)$	$IV(10)$	$IV(30)$
		LHV = SPX xrt		
beta	0.04	−0.12	−0.05	0.10
t-stat	(0.24)	(−0.83)	(−0.36)	(0.63)
$\overline{R}^2$	−0.23	0.98	−0.16	0.51
		LHV = 10yr Bond xrt		
beta	0.07	0.18	0.13	0.02
t-stat	(0.60)	(1.97)	(1.39)	(0.25)
$\overline{R}^2$	0.13	2.00	1.34	−0.36

$$xrt_{t+3} = \alpha + \beta IV_t + \epsilon_{t+3}$$

Univariate IV-predictability: 12-month forecasting horizon

	$IV(SPX)$	$IV(5)$	$IV(10)$	$IV(30)$
		LHV = SPX xrt		
beta	0.12	−0.03	0.07	0.23
t-stat	(1.14)	(−0.20)	(0.59)	(3.02)
$\overline{R}^2$	1.02	−0.36	0.01	4.76
		LHV = 10yr Bond xrt		
beta	0.02	0.20	0.20	0.08
t-stat	(0.21)	(2.20)	(1.69)	(0.82)
$\overline{R}^2$	−0.37	3.69	3.73	0.28

$$xrt_{t+3} = \alpha + \beta IV_t + \epsilon_{t+3}$$

Univariate VRP-predictability: 3-month forecasting horizon

	$VRP(SPX)$	$VRP(5)$	$VRP(10)$	$VRP(30)$
		LHV = SPX xrt		
beta	-0.26	0.13	0.04	-0.21
t-stat	(-3.40)	(1.06)	(0.33)	(-1.92)
$\overline{R}^2$	6.23	1.35	-0.26	4.18
		LHV = 10yr Bond xrt		
beta	0.05	-0.19	-0.14	0.02
t-stat	(0.59)	(-2.34)	(-1.63)	(0.33)
$\overline{R}^2$	-0.16	3.34	1.48	-0.33

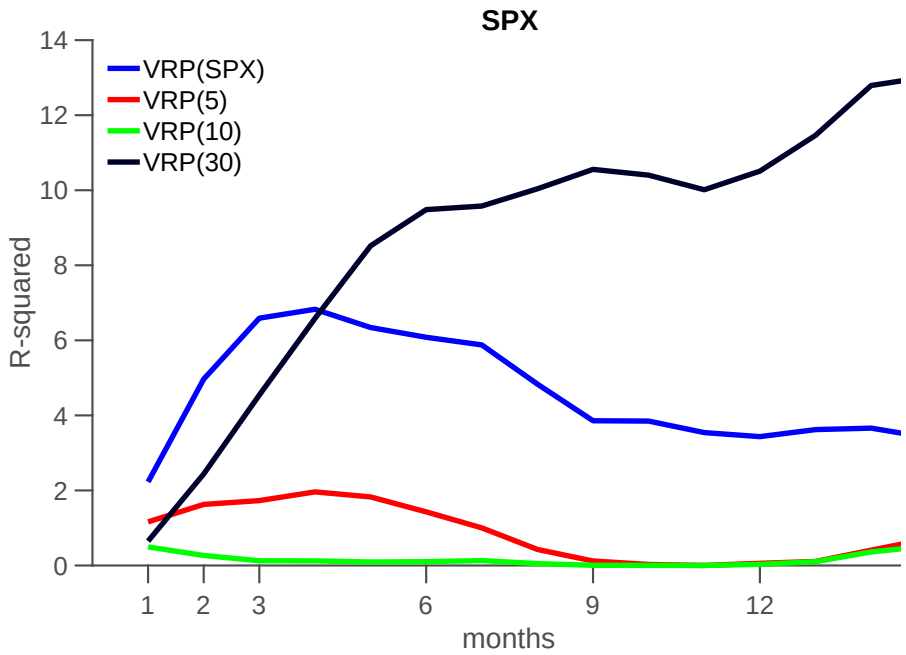
$$xrt_{t+3} = \alpha + \beta IV_t + \epsilon_{t+3}$$

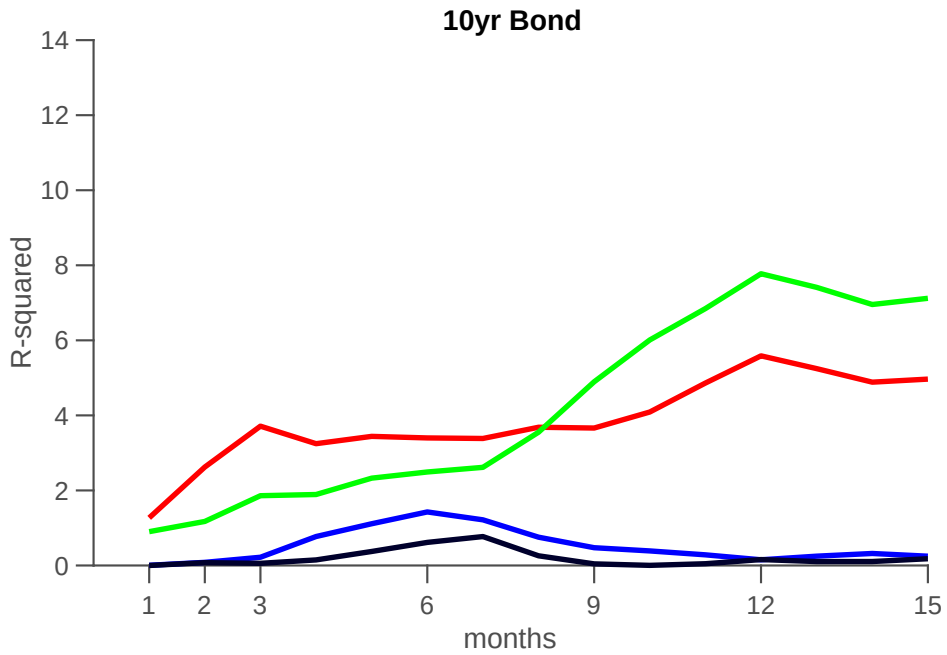
Univariate VRP-predictability: 12-month forecasting horizon

	$VRP(SPX)$	$VRP(5)$	$VRP(10)$	$VRP(30)$
		SPX xrt		
beta	-0.19	-0.04	-0.03	-0.32
t-stat	(-1.80)	(-0.35)	(-0.27)	(-3.63)
$\overline{R}^2$	3.40	-0.25	-0.31	10.07
		10yr Bond xrt		
beta	0.05	-0.23	-0.27	-0.04
t-stat	(0.26)	(-2.37)	(-2.24)	(-0.42)
$\overline{R}^2$	-0.19	4.74	6.90	-0.22

$$xrt_{t+3} = \alpha + \beta IV_t + \epsilon_{t+3}$$

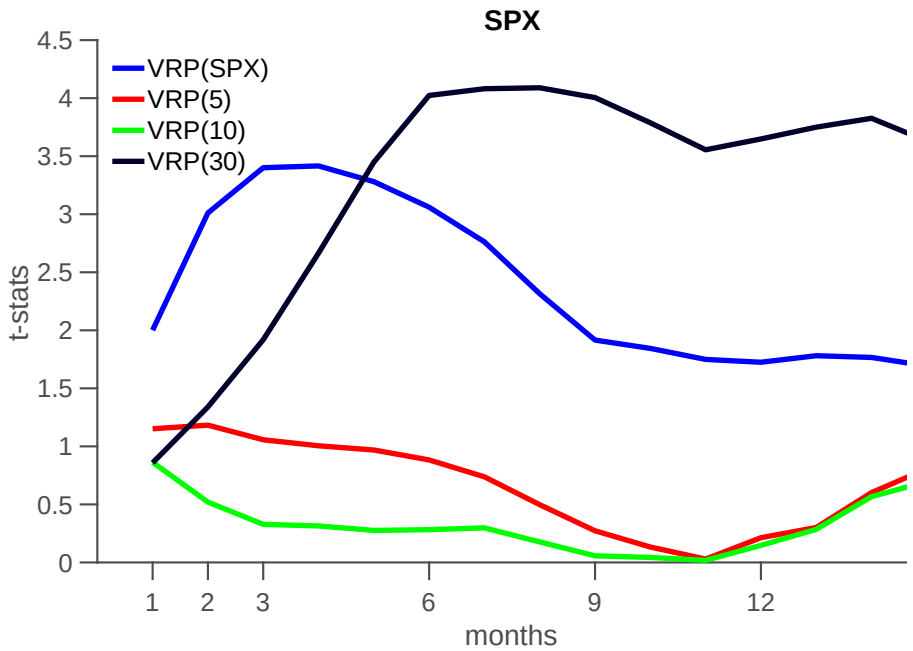
Univariate VRP-predictability: Long horizon $\overline{R}^2$

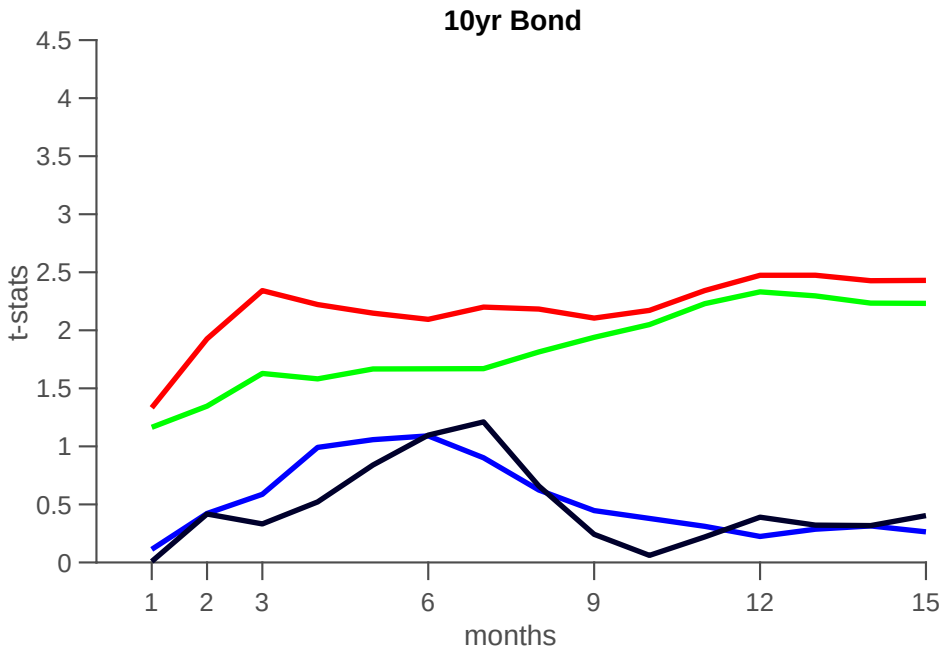




$$xrt_{t+h} = \alpha + \beta VRP_t + \epsilon_{t+h}$$

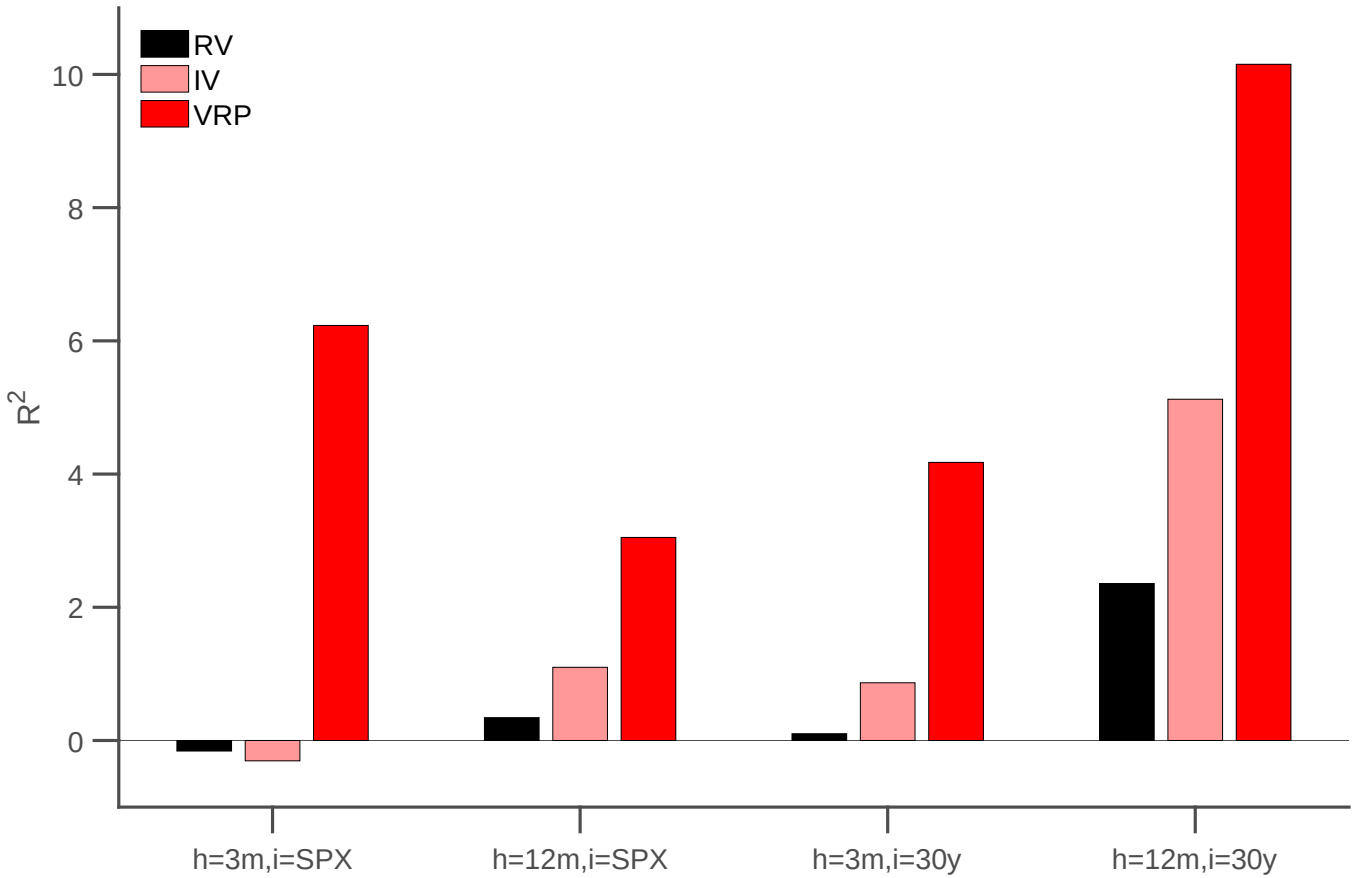
Univariate VRP-predictability: Long horizon t-stats





$$xrt_{t+h} = \alpha + \beta VRP_t + \epsilon_{t+h}$$

Univariate predictability: IV vs VRP



$$LHV = SPX$$

$$RHV = VRP(SPX) \text{ or } VRP(30Y)$$

Questions

Q3 Is there a link between the joint dynamics of expected returns across stock and bonds, and compensation for variance risk?

Treasury VRP Spread: 3-month equity return forecasts

	(i)	(ii)	(iii)	(iv)
$Spread(5)$	-0.32		-0.30	
	(-3.37)		(-3.43)	
$Spread(10)$		-0.36		-0.35
		(-3.49)		(-3.74)
$VRP(SPX)$			-0.23	-0.25
			(-3.01)	(-3.36)
$\overline{R}^2$	10.58	8.69	15.55	14.38

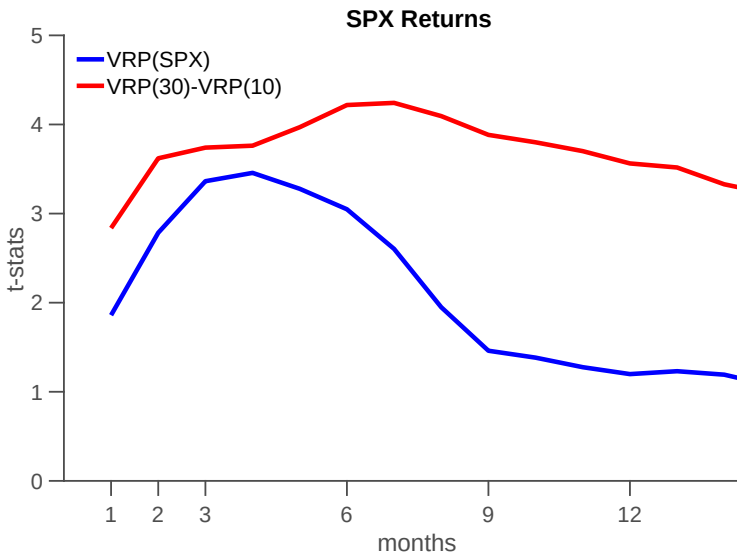
$$Spread(5) = \overline{VRP(30y)} - \overline{VRP(5y)}$$
$$Spread(10) = \overline{VRP(30y)} - \overline{VRP(10y)}$$

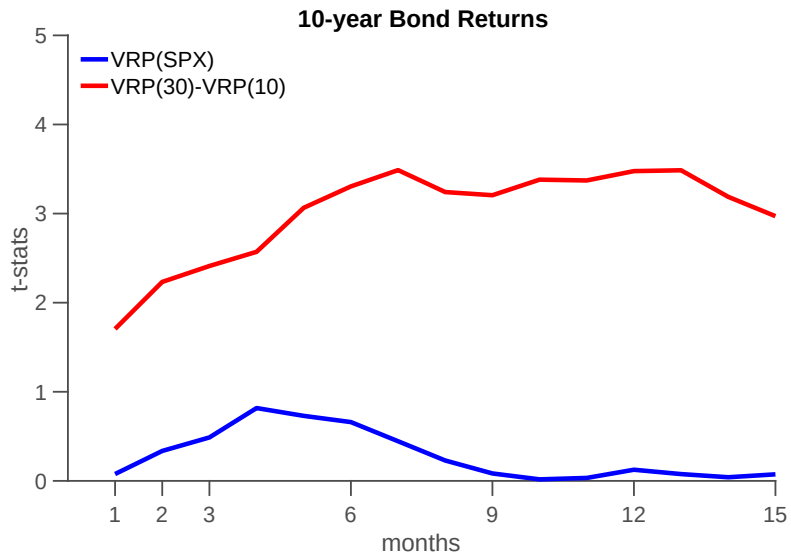
Treasury VRP Spread: 3-month 10-year bond return forecasts

	(i)	(ii)	(iii)	(iv)
$Spread(5)$	0.20		0.20	
	(2.56)		(2.50)	
$Spread(10)$		0.23		0.23
		(2.41)		(2.41)
$VRP(SPX)$			0.03	0.04
			(0.35)	(0.49)
$\overline{R}^2$	3.95	3.39	3.66	3.17

$$Spread(5) = \overline{VRP(30y)} - \overline{VRP(5y)}$$
$$Spread(10) = \overline{VRP(30y)} - \overline{VRP(10y)}$$

Treasury VRP Spread: Long Horizon test statistics

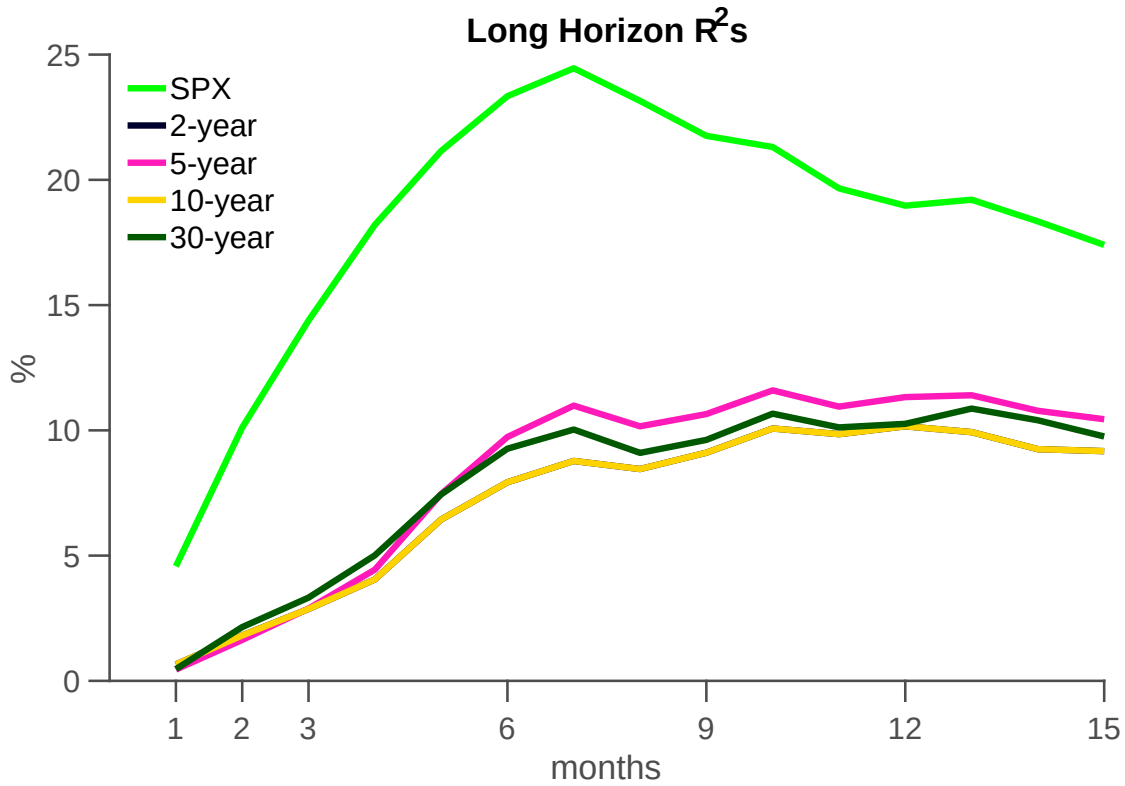




$$xrt_{t+h} = \alpha + \beta_1 Spread + \beta_2 VRP(SPX) + \epsilon_{t+h}$$

$$Spread(10) = \overline{VRP(30y)} - \overline{VRP(10y)}$$

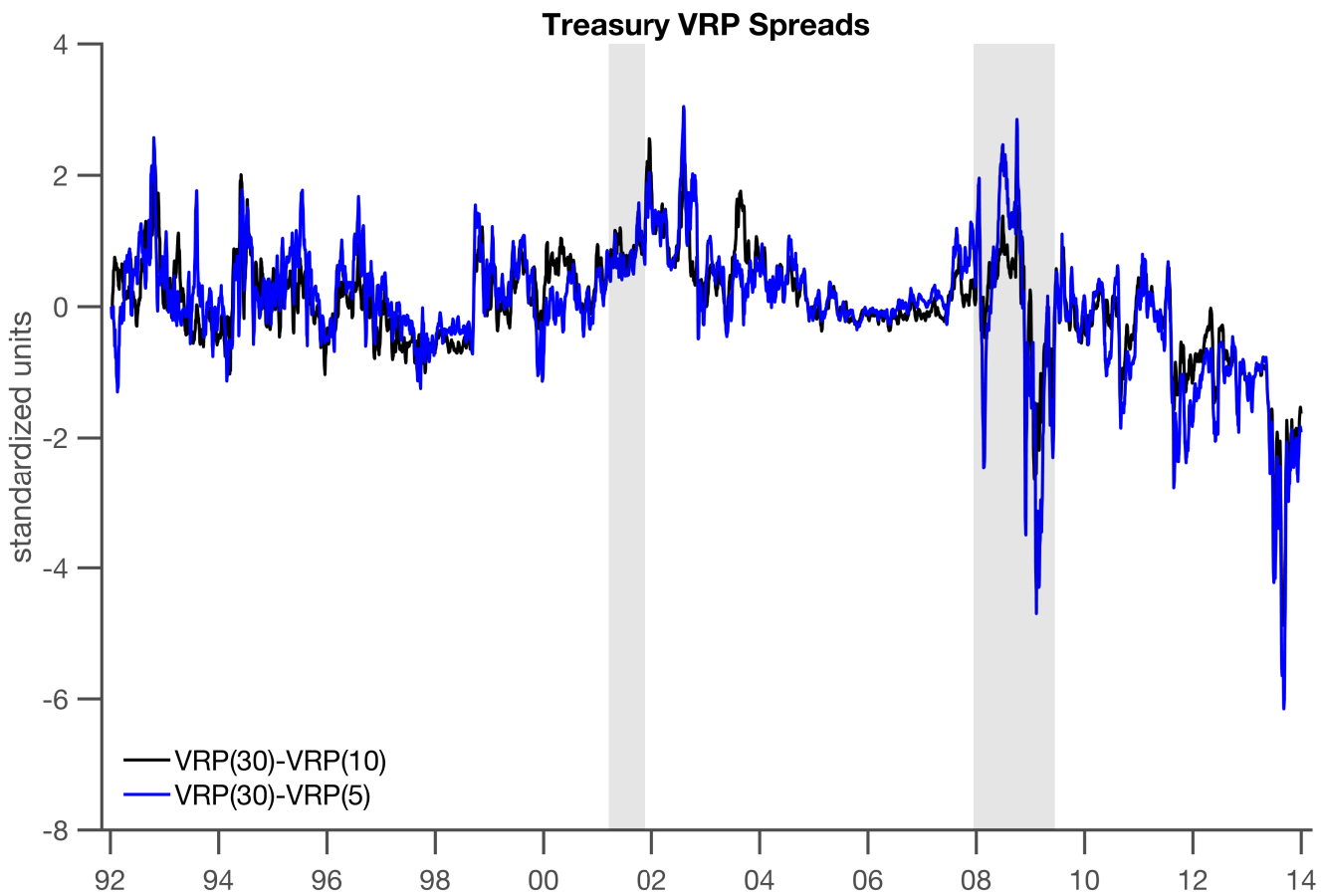
Treasury VRP Spread: Long Horizon test statistics



$$xrt_{t+h} = \alpha + \beta_1 Spread + \beta_2 VRP(SPX) + \epsilon_{t+h}$$

$$Spread(10) = \overline{VRP(30y)} - \overline{VRP(10y)}$$

Treasury VRP Spreads



Questions

Q4 Could consumption based models be consistent with the joint dynamics of compensation for variance risk and expected returns?

Treasury VRP Spreads

	(i)	(ii)	(iii)	(iv)	(v)
Risk Free	0.55				0.46
	(6.18)				(6.66)
Slope	0.54				0.67
	(7.49)				(9.95)
DP		−0.22			−0.46
		(−2.97)			(−7.47)
Cons Surplus			0.11		0.10
			(1.43)		(1.46)
CAY				0.27	0.12
				3.10	(1.43)
$\overline{R}^2$	25.17	4.82	1.27	7.45	39.79

Treasury VRP Spreads (cont.)

	(i)	(ii)	(ii)
Leverage	0.28		0.27
	(3.35)		(3.30)
TED		−0.08	−0.07
		(−1.17)	(−0.87)
$\overline{R}^2$	7.76	0.70	7.51

Leverage is the financial intermediary leverage ratio of He, Kelly, and Manela (2016).

Conclusions

1. **Properties:**
   - ⋆ Bond market variance risk premia are large with is strongly priced with volatility adjusted VRP comparable to equity: $5\text{yr} < 10\text{yr} \sim 30\text{yr} \sim$ Equity
2. **Predictability:**
   - ⋆ neither realised or implied variances are strong related to expected returns
   - ⋆ variance risk premia are strongly related to expected returns and a single spread factor between VRP on long and short dated bonds summarises the information
3. **Economics:**
   - ⋆ We show the Treasury variance spread is related to (a) factors that proxy for consumption risk; (b) the leverage ratio of financial intermediaries.